The International Library of Philosophy

THE LIMITS OF SCIENCE

ARBOR SCIENTIÆ

ARBOR VITÆ

Founded by C. K. Ogden

The International Library of Philosophy

THE LIMITS OF SCIENCE

Founded by C. K. Ogden

The International Library of Philosophy

PHILOSOPHY OF LOGIC AND MATHEMATICS
In 8 Volumes

I	The Nature of Mathematics	*Black*
II	The Technique of Controversy	*Bogoslovsky*
III	The Limits of Science	*Chwistek*
IV	Foundations of Geometry and Induction	*Nicod*
V	The Foundations of Mathematics	*Braithwaite*
VI	Logical Studies	*von Wright*
VII	A Treatise on Induction and Probability	*von Wright*
VIII	An Examination of Logical Positivism	*Weinberg*

THE LIMITS OF SCIENCE

Outline of Logic and of the Methodology of the Exact Sciences

LEON CHWISTEK

Introduction and Appendix by
Helen Charlotte Brodie

Routledge
Taylor & Francis Group
LONDON AND NEW YORK

First published in 1948 by
Kegan Paul, Trench, Trubner & Co Ltd

Published 2014 by Routledge
2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN
711 Third Avenue, New York, NY, 10017, USA

Routledge is an imprint of the Taylor & Francis Group, an informa business

British Library Cataloguing in Publication Data
A CIP catalogue record for this book
is available from the British Library

ISBN 13: 978-0-415-22544-1 (hbk)
ISBN 13: 978-0-415-61418-4 (pbk)

" They say miracles are past ; and we have our philosophical persons, to make modern and familiar, things supernatural and causeless."

WILLIAM SHAKESPEARE, *All's Well that Ends Well*, Act ii, Scene 3.

This page intentionally left blank

TABLE OF CONTENTS

AUTHOR'S PREFACE TO ENGLISH EDITION

The English edition of *Granice Nauki* is essentially different from the original text.

Chapter VII is completely changed. In Chapters VIII and IX important additions have been made.

I am greatly indebted to Miss Brodie, Mgr. Herzberg, and Dr. Hetper for important critical remarks.

LEON CHWISTEK.

Lwów.

This page intentionally left blank

TRANSLATORS' PREFACE

Significant contributions have been made by contemporary Polish thinkers in the fields of logic, philosophy of science, and the analysis of the foundations of mathematics. They have been the initiators and leaders of contemporary thought on many important issues involved in metalogic and linguistics. Relatively little of their work is available in any international language and hence the writings of these theoreticians have remained comparatively unfamiliar to students in these fields in other countries of Europe and in America.

The Limits of Science, by Dr. Leon Chwistek, was first published in 1935 under the title *Granice Nauki*. The present edition has been revised and supplemented by the author. The translators are indeed grateful to Dr. Chwistek for his whole-hearted co-operation both in revising the work and carefully checking the manuscript.

Taking the views of Bertrand Russell, Henri Poincaré, and David Hilbert as his point of departure, Dr. Chwistek goes on to develop rational semantics, which he contends can be successfully applied in solving the problems which arise in connection with philosophy, science, social theory, and art. *The Limits of Science* is the culmination of Dr. Chwistek's thought with regard to the application of rational semantics to logic, the philosophy of mathematics, and the foundation problems of the physical sciences.

It is perhaps unnecessary to point out, especially to the Polish reader, that the translators have directed their efforts toward a free translation rather than a word-for-word rendering of the text. It has proved more feasible to eliminate certain idiomatic expressions of the Polish language and allusions familiar only to the Polish reader, and to concentrate our efforts upon obtaining an adequate and coherent interpretation of the text. Wherever possible translations of quotations from works in foreign languages have been taken directly from the English translation of these works.

The translators wish to express their thanks to Professor Herbert W. Schneider who initially encouraged this project; Professors Haskell B. Curry, A. F. Bentley, and Rudolf Carnap, whose recognition of its value motivated its execution; Dr. J. Herzberg, of Lwów, who gave invaluable

assistance in the tedious task of checking references ; Professor Horace L. Friess, who gave freely of his time in checking references, interpreting allusions, and discussing certain problems which arose in connection with the translation ; Dr. H. Theodric Westbrook and Dr. Ernest Moody, who willingly offered suggestions and criticisms in rendering quotations taken from the medieval Latin ; to Dr. Josef Maier, who verified translations of quotations taken from German authors, and to Miss Jean Macalister, of the Columbia University Library, who checked several obscure references.

The translators are deeply indebted to Professor Ernest Nagel without whose efforts the publication of this translation would have been impossible. He not only undertook to make the initial arrangements for publication, offered his advice with regard to the problems which arose in connection with the work, but checked the manuscript in its entirety, offering invaluable suggestions and criticisms with regard to terminology and interpretation.

In offering this translation of *The Limits of Science* to the philosophical public it is the hope of the translators that this initial translation of a logical text from Polish into English will not be the last, that an increase in the familiarity of Western thinkers with the works of Polish theoreticians written in their native tongue will follow and that a more adequate understanding and evaluation of their contributions will be obtained.

H. C. B.
A. P. C.

PREFACE TO INTRODUCTION AND APPENDIX

Chwistek's views on logic, and in particular those concerning semantics and metamathematics, were developed over a period of many years. However a study of his writings of the last four or five years reveals that, except for matters of detail, his views have attained their final form. For this reason it is important to indicate explicitly how the present text differs in form from the original edition. Chwistek himself points out that:

> "In Chapters IV–VI instead of the Greek letters α, β, . . . the letters *u*, *v*, *w*, . . . are employed. Otherwise there will be no conformity with the system of Chapter VII. Symbols such as **(0000)**, etc., have no individual meaning; they are not names at all. To have significant propositions we must assume that **(0000)** *is true*, or that it is a theorem."

In a series of letters written during the summer of 1939 Chwistek dealt specifically with the varieties of type to be used in setting up the manuscript and submitted certain general directions, which can be summarized as follows:

1. Italics are to be employed in the case of sentences of the symbolic language, and in the case of real and apparent variables (both logical and semantical) which are not starred expressions. They are also to be used to indicate phrases or sentences which are emphasized.
2. Bold face is to be employed in the case of the language of interpretation *and* the interpreted language. Constant expressions, logical operators, and variables (whether real or apparent), which are defined as starred expressions, are also to be printed in bold face.
3. All mathematical symbols, when not considered within the context of the system of semantics or metamathematics, are to be written in accordance with the usual mathematical conventions.

The application of these directions was left in all cases to the present writer. Unfortunately many questions of interpretation arose in this connection and Dr. Chwistek was unable to read the final draft of the manuscript in which they were resolved. While Professor Ernest Nagel aided immeasurably in dealing with them, the actual responsibility for the choice of type of all symbols must rest upon the writer.

It might be added here that in any case it would have been impossible to follow the typography of the original edition not only because of fundamental changes in Chwistek's position since its publication in 1935, but because of the inclusion in the text of a considerable amount of hitherto unpublished material.

In the Introduction and Appendix an attempt is made to develop a consistent interpretation of Chwistek's views, to eliminate all their " obscurities ", and to give an adequate evaluation of them. It is therefore necessary to employ terminology current among other logicians as well as Chwistek's own phraseology. Consequently on occasion deviations from Chwistek's terminology and notation may be found on certain fundamental points.[1] Quotation marks, for example, are employed to indicate the name of an expression. Although Chwistek himself does not accept this convention, it readily permits the reader to discover exactly what Chwistek has in mind at a given point. While this and other reformulations employed in the Introduction and Appendix have been given only after a careful consideration of Chwistek's views in his own terms, they are essential if his position is to be understood and evaluated by other logicians. Since, however, Chwistek never saw these portions of the text it is impossible to decide whether he would be willing to accept the writer's interpretation of his views exactly as they stand. The reader can test its adequacy by an examination of the translation itself, where Chwistek's own symbolism and notation remain unchanged.

Finally the writer wishes to express her appreciation to Wellesley College, under whose auspices the Introduction and Appendix were completed during her term as Alice Freeman Palmer Fellow (1939–1940).

H. C. B.

[1] However in actual quotations from Chwistek's writings, all his conventions (past and present) are followed, except where specifically indicated. All translations from the Polish were made by the writer.

BIBLIOGRAPHY OF CHWISTEK'S ARTICLES AND BOOKS

To aid the reader in looking up references made in the text an abbreviation has been assigned to items mentioned in the text, introduction and appendix, and the items listed alphabetically rather than chronologically. If an article has been written in collaboration with other writers this fact is indicated.

A.L.F. . . . " Antynomje logiki formalnej " (" Antinomies of Formal Logic "), *Przegląd filozoficzny*, vol. 24, 1921, pp. 164–171.

" Sur les fondements de la logique moderne," *Atti del V Congresso Internazionale di Filosofia Napoli 5–9 Maggio 1924*, Naples, 1925, pp. 24–8.

F.M.R. . . . " Podstawy metamatematyki racjonalnej " (" Fondements de la métamathématique rationelle ") *Bulletin International de l'Académie Polonaise des Sciences et des Lettres*, Classes des sciences mathématiques et naturelles, Serie A, sciences mathématiques, Kraków, 1933, pp. 253–264, written in collaboration with W. Hetper and J. Herzberg.

F.P.G.T. . . " A Formal Proof of Gödel's Theorem," *The Journal of Symbolic Logic*, vol. 4, no. 2, 1939, pp. 61–8.

F.S.E. . . . " Sur les fondements des sciences exactes," F. Gonseth : *Philosophie mathématique*, pp. 88–91, Actualités scientifiques et industrielles, 837, Philosophie chronique annuelle publiée par l'institut international de collaboration philosophique, IV, Paris, 1939.

G.N. . . . . *Granice Nauki, Zarys logiki i metodologji nauk ścisłych* (*The Boundaries of Science, Outline of Logic and Methodology of the Exact Sciences*), Lwów and Warsaw, 1935, xxiv and 264 pp. The original of this work.

" Infinitely Small Numbers and Their Application," *The Journal of Unified Science* (*Erkenntnis*), vol. ix, pp. 153–8.

" Krotka rozprawa z panem Romanem Ingardenem doktorem uniwersytetu fryburskiego" (" Brief Discussion with Mr. Roman Ingarden, Doctor of the University of Freiburg "), *Przegląd filozoficzny*, vol. 25, 1922, pp. 541–4.

K.P.Z. . . . " Krytyka pojęcia zmiennej w systemie semantyki racjonalnej " (" A Critique of the Concept of the Variable in the System of Rational Semantics "), *Archiwum towarzystwa naukowego we Lwowie*, Dział III, matematyczno-przyrodniczy, Tom IX, 1938, pp. 283–334.

" La lutte contre l'idéalisme," *Congrès International de Philosophie Scientifique, Paris, 1935*, Actualités scientifiques et industrielles no. 388, 1936, i, pp. 79–80.

" Liczby nieskończenie małe i ich zastosowanie " (" Infinitely Small Numbers and Their Applications "), *Sprawozdania towarzystwa naukowego we Lwowie*, vol. 18, 1938, pp. 336–8.

L.R. . . . . " Logisches zur Relativitätstheorie," *Congrès International de Philosophie Scientifique, Paris, 1935*, Actualités scientifiques et industrielles, no. 392, 1936, pp. 54–5.

La Méthode Générale des Sciences Positives; L'Esprit de la Sémantique. Actualités Scientifiques et Industrielles no. 850, Logique et Méthodologie, Exposés publiés sous la direction de Thomas Greenwood, VI. (Hermann et Cie., Paris, unpublished.)

M.L. . . . " Miara Lebesgue'a. Logiczna analiza i konstrukcya pojęcia miary Lebesgue'a " (" Lebesgue Measure. Logical Analysis and Construction of the Concept of Lebesgue Measure"), *Archiwum Towarzystwa Naukowego we Lwowie,* Wydział III, matematyczno-przyrodniczy, vol. 2, 1922–3, pp. 111–121.

M.M.A. . . " Une méthode métamathématique d'analyse," *Sprawozdanie z I Kongresu Matematyków Krajów Słowiańskich (Comptes Rendus du I Congrès des Mathematiciens des Pays Slaves), Warszawa, 1929,* Warsaw, 1930, pp. 254–263.

N.F.F.M. . . " New Foundations of Formal Metamathematics," *The Journal of Symbolic Logic,* vol. 3, no. 1, 1938, pp. 1–36, written in collaboration with W. Hetper.

N.G.L.M. . . " Neue Grundlagen der Logik und Mathematik," *Mathematische Zeitschrift,* Bd. 30, 1929, pp. 704–724.

N.G.L.M.II. . " Neue Grundlagen der Logik und Mathematik, Zweite Mitteilung," *Mathematische Zeitschrift,* Bd. 34, 1932, pp. 527–534.

N.G.M. . . " Die Nominalistische Grundlegung der Mathematik," *Erkenntnis,* Bd. 3, 1932–3, pp. 367–388.

" Nouvelles recherches sur les fondements des mathématiques," *Atti del Congresso Internazionale dei Matematici, Bologna 3–10 Settembre, 1928 (VI),* vol. 3, Bologna, 1930, pp. 375–380.

" O granicach nauki " (" Concerning the Boundaries of Science "), *Sprawozdania poznańskiego towarzystwa przyjaciól nauk,* vol. 9, 1935, pp. 9–10.

P.O.T.K. . . " Podstawy ogólnej teorji klas " (" Foundations of the General Theory of Classes "), unpublished.

" Podstawy ogólnej teorji klas " (" Foundations of the General Theory of Classes "), *Sprawozdania towarzystwa naukowego we Lwowie,* vol. 17, 1937, pp. 247–9.

P.P.R.S. . . " Kilka uwag o podstawowych prawach rozchodzenia się światła " (" Quelques remarques sur les lois fondamentales de la propagation de la lumière "). *Archiwum towarzystwa naukowego we Lwowie,* Dział III matematyczno-przyrodniczy, tom 9, 1937, pp. 247–252.

" Pluralité des réalités," *Atti del V Congresso Internazionale di Filosofia, Napoli 5–9 Maggio, 1924,* Naples, 1925, pp. 19–24.

" Rola semantyki racjonalnej w filozofii. Filozoficzne znaczenie logiki semantycznej." ("The Role of Semantics in Philosophy. Philosophical Significance of Semantical Logic.") *Przegląd filozoficzny,* vol. 39, 1936, pp. 331–4.

" Role zasady konsekwencyi w zagadnieniu sprawiedliwości społecznej " (" Role of the Principle of Consistency in the Problem of Social Justice "), *Przegląd filozoficzny,* vol. 39, 1936, pp. 494–9.

" La sémantique rationelle et ses applications," *Travaux du IXe Congrès international de philosophie,* Actualités scientifiques et industrielles, no. 535, VI, 1937, pp. 77–81.

T.C.T., T.C.T. II. " The Theory of Constructive Types. Principles of Logic and Mathematics." *Rocznik Polskiego Towarzystwa*

Matematycznego (*Annales de la Société Polonaise de Mathématique*), vol. 2, 1924, pp. 9–48, vol. 3, 1925, pp. 92–141.

T.L. . . . " Trzy odczyty odnoszące się do pojęcia istnienia " (" Three Lectures relating to the Concept of Existence ") *Przegląd filozoficzny*, vol. 20, 1917, pp. 122–151.

T.V.M. . . " Tragedja werbalnej metafizyki" ("The Tragedy of Verbal Metaphysics ") *Kwartalnik filozoficzny*, vol. x, Kraków, 1932, pp. 46–76.

U.A.P.M. . . " Über die Antinomien der Prinzipien der Mathematik," *Mathematische Zeitschrift*, Bd. 14, 1922, pp. 236–243.

U.H.M. . . " Über die Hypothesen der Mengenlehre," ibid. Bd. 25, 1926, pp. 439–473.

U.B. . . . " Überwindung des Begriffsrealismus," *Studia Philosophica*, vol. 2, 1937, pp. 1–18.

" Uwagi o podstawach metamatematyki racjonalnej " (" Remarques sur la méthode de la construction des notions fondamentales de la métamathématique rationelle ") *Bulletin International de l'Académie Polonaise des Sciences et des Lettres*, Classes des sciences mathématiques et naturelles, Série A, sciences mathématiques. Kraków, 1933, pp. 265–275, written in collaboration with W. Hetper and J. Herzberg.

" W kwestji zdań ' pozbawionych treści '. Z powodu polemiki o definicję wielkości." (" On the Question of Propositions ' Without Content '. As a consequence of the Polemic concerning the Definition of Greatness "), *Przegląd filozoficzny*, vol. 22, 1919, pp. 110–11.

W.R. . . . Wielość Rzeczywistości (*The Plurality of Realities*), Kraków, 1921, 96 pp.

" Zasady czystej teorji typów " (" Principles of the Pure Theory of Types "), *Przegląd filozoficzny*, vol. 25, 1922, pp. 359–391, p. 564.

" Zasady czystej teorji typów " (" Principles of the Pure Theory of Types "), *Przegląd filozoficzny*, vol. 27, 1927, pp. 34–6.

Z.K.D.P. . . *Zagadnienia kultury duchowej w Polsce* (*The Problems of Intellectual Culture in Poland*), Warszawa, 1933.

Z.M.C. . . " Zastosowanie metody konstrukcyjnej do teorji poznania " (" The Application of the Constructive Method to the Theory of Knowledge "), *Przegląd filozoficzny*, vol. 26, 1923, pp. 175–187.

" Zastosowanie metody konstruktywnej do teorji poznania " (" The Application of the Constructive Method to the Theory of Knowledge "), *Przegląd filozoficzny*, vol. 27, 1927, pp. 296–8.

Z.S. . . . " Zasada sprzeczności w świetle nowszych badań Bertranda Russella " (" The Principle of Contradiction in the Light of Recent Investigations of Bertrand Russell "), *Rozprawy Akademii Umiejętności*, Kraków, Wydział historyczno-filozoficzny, 2 s. vol. 30, 1912, pp. 270–334.

This page intentionally left blank

INTRODUCTION

For almost three decades Dr. Leon Chwistek has been occupied with the problems of philosophy and logic. While it is generally recognized that he has made valuable contributions to logic by his critical analysis of its foundations, his own doctrines in philosophy and logic have been the subject of heated debate, particularly in his native land. *The Limits of Science* presents Chwistek's position on some of these controversial issues. While admittedly but an outline of the methodological and logical problems of the exact sciences, this work merits consideration as an attempt to approach these problems from the standpoint of a new logical science, semantics, and a new view concerning reality, the theory of plural reality. In the light of current philosophical discussion it is of interest to note that Chwistek was led to work out this new approach as a result of a prior analysis of language. His position can therefore be characterized as the reaction of a present-day nominalist to contemporary realistic and anti-rationalist doctrines.

Unfortunately, however, Chwistek uses a vocabulary and symbolic apparatus different from that of other philosophers and logicians. A number of writers have tried to restate his views in more familiar terminology. Consequently differences of opinion have arisen concerning the proper interpretation of his position as well as its validity. Chwistek has defended his position largely by attacking that of his opponents. But just as Chwistek's critics have never really tried to understand his views in *his* terms, he has never really tried to understand the viewpoint of his critics. An adequate evaluation of his point of view requires that some common basis of discussion be attained. It is the aim of this introduction to supply this lack and to contribute to such an evaluation. An attempt will therefore be made to place Chwistek's logical views in their proper philosophical setting. The influence of his logical theories upon more general philosophical considerations will also be examined.

I

The Method of Sound Reason

The Criteria of Sound Reason

It is a commonplace to state that philosophers and scientists both seek a coherent conception of the world and attempt to give an adequate analysis of experience. In view of this identity of aim it is not entirely surprising that their views have had an influence upon each other. Nevertheless the scientist and philosopher treat different aspects of experience. The physicist for example may be concerned with the conditions for the occurrence of electrodynamic phenomena, while the philosopher may be concerned with the generalized problems of knowledge.

It is Chwistek's contention, as a result of long preoccupation with philosophic and scientific problems, that certain weaknesses inherent in scientific procedure have given rise to many false philosophic doctrines. For example the inability of the Greek philosophers to remove the paradoxes discovered by Zeno (192 ff.)[1] gave rise to a philosophy of " pure being ". Thus in times of crisis in the history of science philosophers have been wont to advance doctrines in which exact thought is replaced by vision and phantasy. In this way Chwistek explains the widespread influence of Plato's thought. He interprets Hegel's views as arising from the confusion among eighteenth and nineteenth century mathematicians concerning the nature of infinitesimals.

Chwistek himself protests against any philosophic doctrine which is based upon " absolutes ", because they cannot be exemplified in or verified by experience. For this reason he objects to such concepts as " the perfect good " of Socrates, the " ideas " of Plato, the " absolute truth " of Hegel, and the " absolute knowledge " of Husserl. Philosophic doctrines, he maintains, are to be secured by the application of reason (i.e. sound reason) to experience. Only in this way is it possible to attain knowledge and add to the scope of our experience.[2]

Chwistek's analysis of sound reason has obviously been

[1] References to the text will be inserted in parenthesis.

[2] Chwistek also maintains that metaphysical elements must be eliminated from science. He objects, for example, to the introduction of entelechies into biology by Driesch (5). Nevertheless he insists that some method must be found to eliminate such scientific puzzles as the already mentioned paradoxes of Zeno. Once again Chwistek has recourse to the method of sound reason.

motivated by the situation in philosophy where philosophers, who present utterly incompatible views, each claim to have achieved knowledge. Chwistek himself believes that it is possible to discover certain fundamental truths, which though perhaps trivial, are not subject to variations in interpretation. These propositions, which are open neither to serious dispute nor alteration, he regards as the foundation of science and philosophy. Chwistek clearly formulates his view, when he defines sound reason as " the method for attaining truths not subject to intellectual revolution " (25).

Unfortunately, however, it is difficult to discover exactly what Chwistek means by sound reason. While he recognizes that it consists of a number of fundamental assumptions,[1] he asserts that " its criteria cannot be formulated in a pattern " (265), and that these criteria are variable (płynny).[2] He freely admits that " the exact bounds of their operation cannot be fixed " (265). The net result of his discussion of sound reason is therefore merely a statement of some of the well-known features of the reflective method, although these features do not characterize this method completely.

Chwistek has selected for consideration various criteria of the method of sound reason. But in the case of each of these criteria he recognizes its inadequacy as a defining characteristic of this method. Sound reason, for example, relies upon habits, but habits are subject to alteration. Again, sound reason works successfully only in the domain of familiar phenomena. Even the laws of thought, which are also advanced as positive criteria of this method, are subject to these limitations (29–30).[3] Nor can Occam's razor (43) guarantee reliable knowledge. As a rule of selection, which requires the acceptance of the simpler of two alternative explanations, it has a negative role ; but even in this capacity this principle cannot be formulated precisely.

[1] Contrary to the procedure of certain present day philosophers, Chwistek admits the dependence of his views upon certain assumptions. Cf. e.g. *Z.M.C.*, p. 186, " . . . they " [the philosophers] " forget only too often that the demonstration of anything requires the acceptance of some supposition . . . the acceptance of suppositions is an arbitrary act and is . . . conditioned by a certain feeling of truth which, however, is undoubtedly subjective and cannot be forced upon any one as necessary."

[2] *W.R.*, p. 46.

[3] Chwistek points out the validity of the principle of contradiction with respect to definite questions which require definite answers. But he also points out the necessity of specifying certain supplementary and frequently artificial conditions in the case of propositions involving change (29–30). Cf. also *Z.S.*, p. 276.

Chwistek also characterizes sound reason as a method which involves criticism. But while he realizes the importance of being aware of the function of reason in science and philosophy he also recognizes the part played by the emotions, intuition, and background of the scientist and philosopher in the development of their views. For this reason the method of sound reason cannot be identified solely with criticism. For the exercise of sound reason requires not only criticism but " construction ". Unfortunately Chwistek's usage of the latter term is not free from ambiguity. In his treatment of the natural sciences and the problem of reality he uses this term as a synonym for the synthesis of concepts. In the case of the deductive sciences he evidently has in mind the construction of systems.

Chwistek's consideration of these characteristics of the method of sound reason shows that none of them formulate the method adequately. Each of these characteristics must be regarded as referring only to a partial method, whose application in conjunction with other such partial methods constitutes an application of the method of sound reason. A criterion for the failure to use the method of sound reason in some particular analysis, according to Chwistek, is that one of these partial methods has not been employed. For example, he regards Hegel's doctrines as anti-rational [1] because they are incompatible with one of the fundamental principles of sound reason, the principle of contradiction (12–14). Many other citations might be offered in support of this interpretation of the method of sound reason.

[1] Chwistek uses the word " anti-rational " together with the terms " metaphysical ", " idealistic ", and " fictional " as derogatory epithets. A doctrine is " anti-rational " if it is not obtained by the application of the method of sound reason. The terms " metaphysical ", " idealistic ", and " fictional " are used to refer to concepts which have no experiential base and consequently cannot be verified by reference to experience. For example, Chwistek regards Newton's absolute space as a metaphysical, idealistic, and fictional concept. This general position is familiar to the reader of contemporary positivistic literature. It should be noted, however, that Chwistek extends the usual list of terms of opprobrium far beyond its usual length.

It is of course possible to quarrel with Chwistek's terminology since he assigns new meanings to familiar philosophical terms. However his general intent is clear enough. It is therefore important to point out that Chwistek does not feel that problems which are usually called metaphysical are either meaningless or idealistic, when conceived as the study of the fundamental problems of existence. He does not for example hold that the problem of the relation between the soul and body is meaningless (cf. *W.R.*, pp. 39–40). He discusses the problem of free will at some length (*W.R.*, pp. 40–1, 54–5). This recognition of the possibility of metaphysics clearly distinguishes his views from those of many present-day positivists.

Language in the Light of Sound Reason

The formulation of the results of scientific and philosophical research in purely linguistic terms necessitates a language suitable for precise investigations, to be used in conformity with the principles of sound reason. Consequently Chwistek attempts to formulate some of the criteria of meaningful discourse. In the main he follows the British empirical tradition in identifying the meanings of a term with the ideas or images evoked by it. Accordingly, when " meaning " is so conceived, a term will vary considerably from individual to individual and from situation to situation.[1] It is therefore not difficult for Chwistek to show the falsity of the view that concepts have an absolute " real " meaning, which is the same for all individuals. Nor does Chwistek find any merit in the view, advanced by writers such as Husserl, that there are apriori laws for distinguishing the meaningful from the meaningless.

Chwistek then raises the question whether everyday language is an instrument suitable for scientific and philosophical purposes. His answer, which is in the negative, is based largely upon the theory of meaning which he proposed. Everyday language contains many abstractions which are treated as concrete objects. As Shestov says, " Just as things of the external world have a real existence for us, so the good has a real existence for Socrates " (27). Plato regards the soul as an object in everyday use (28). Such general concepts are subject to individual interpretation. Because the same term is used in different meanings in everyday language it is not difficult to construct contradictions in this language (40–2). Leonard Nelson, for example, has uncovered the following paradoxical[2] situation in epistemology (271)[3]:

> Epistemology is concerned with the problem whether or not objective knowledge is possible. To solve this problem it is assumed that there exists some criterion which can be applied in its solution. This criterion must obviously either be knowledge or not.
>
> If this criterion is knowledge it belongs to the domain whose validity is being examined and is therefore problematical.

[1] It is not Chwistek's intent to dispense with any of these meanings nor with any of the terms current in philosophical and scientific discourse. He requires only that the meanings of the terms used be clearly and carefully specified.

[2] A paradox or antinomy is a statement which can be shown to be both true and false on the basis of the same set of premises.

[3] Cf. *W.R.*, pp. 38–9.

> Consequently the criterion to be used in solving the epistemological problem cannot itself be knowledge.
>
> On the other hand if the criterion to be used is not knowledge, then it itself must be known, i.e. the criterion of knowledge must be employed.

While such difficulties may at first sight seem trivial they have far-reaching consequences for an adequate logic and philosophy of mathematics. Since it is possible to construct such contradictions in everyday language, this language is obviously not consistent and the rules which govern its construction and usage cannot guarantee that it will function correctly. It is not then a language which is suitable for scientific and philosophical purposes.

If, however, Nelson's epistemological paradox is examined more closely it will be noted that the paradox is obtained only by using two different senses of the word " knowledge " interchangeably. The word " knowledge " has been made to refer to itself. It has been suggested by some writers that paradoxes can be eliminated by postulating that a concept or statement cannot be used to refer to itself. This suggestion has been worked out in various ways, and the rules proposed for the attainment of this end are called theories of types. Chwistek has made a positive contribution to the theory of meaning [1] by suggesting a theory of types for *everyday language*, with the help of which such contradictions as the epistemological paradox [2] will be eliminated from this language. He has thus outlined a device for preventing the assigning of a single property to different types of entities.

The difficulties which Chwistek finds in everyday language have led him to adopt a position which he calls " nominalism ". For example, his rejection of abstract ideas and universals (xxii, xxv) leads him to maintain that the scientist and philosopher

[1] Chwistek even goes so far as to recognize that the term " meaning " is not itself entirely unambiguous and he suggests the possibility of a hierarchy of propositions formed on the basis of different meanings of the word " meaning ". Cf. *W.R.*, p. 85, and *Z.S.*, p. 330.

[2] On Chwistek's view the epistemological paradox involves the sentence, " The criterion for the epistemological problem is knowledge," and the sentence, " ' The criterion for the epistemological problem is knowledge ' is knowledge." In the first sentence the property of " being knowledge " is predicated of a noun. In the second sentence this property is predicated both of a noun and of a sentence. The property of " being knowledge " when predicated of a noun is of lower type than when predicated of a sentence. It has not the same meaning in both cases. If these facts are realized it becomes impossible to reason in the manner indicated by Nelson and the paradox in question cannot arise.

should be concerned only with concrete objects. While he therefore frowns upon the use of general concepts, individual words, and names play an important role in Chwistek's conception. But because of the ambiguity of everyday language, the various ideas which its terms evoke must be carefully distinguished from one another, and a new and more precise language must be developed in which each idea is represented by a specific symbol (e.g. word, sign, or name). Chwistek's method of sound reason expresses this nominalism. Its application reduces reasoning to the performance of purely mechanical operations upon symbols, analogous to calculatory operations. With the help of this method Chwistek hopes to secure the greatest amount of certainty in knowledge, within the limits of human reason. He insists there is no break in continuity between the kind of knowledge obtained in daily life and the kind obtained in the theoretical and experimental sciences. Consequently scientific propositions themselves are subject to the qualified certainty which the method of sound reason can give ; knowledge of the world can never be complete.[1]

Logic and Sound Reason

Because sound reason critically employed suffers from obvious limitations, Chwistek finds it necessary to supplement the uncontrolled operations of sound reason by a new device which he calls " logic " However he uses the term " logic " in two distinct senses which he himself does not carefully differentiate. In one sense " logic " is taken to be " the basis of all thought " and can be construed as a general methodology. Its function is to distinguish the various categories of experience and to supply fixed rules in accordance with which sound reason may operate. With its help it is possible to differentiate beliefs which are held because of unreflective habit from those supported by reflective thought. It is with logic as a methodological instrument, co-extensive with the method of sound reason, that Chwistek has been concerned up to this point.

In the second and more frequent sense in which Chwistek uses the term, logic is identified with a formal system. A formal system must of course conform to the principles of sound reason already mentioned. However, the specific task of

[1] This fact is easily recognized if the conditions of human knowledge formulated in the criteria of sound reason are understood. Moreover, on the basis of the theory of types a large set of sentences must be regarded as meaningless and consequently cannot be admitted as knowledge.

formal logic embraces the formulation of concepts in terms of an unambiguous and precise symbolism as well as the elimination of appeals to intuition and of "metaphysical" assumptions. Consequently the development of a formal logic involves not only the analysis of the concepts of systems of knowledge already adopted, but their reconstruction on the basis of this analysis. In this way Chwistek hopes to avoid the hypostatizations against which his nominalism is directed, to expose the inadequacies of idealistic, realistic, and anti-rational systems of logic, and to convince the reader of their uselessness. Chwistek himself employs "logic" in this second sense only when he formalizes the mathematical sciences. However, although he does not apply formal logic to the philosophy of science and problem of reality, he offers constructive suggestions concerning these domains based upon his system of logic.

The construction of a system of formal logic [1] is carried through by specifying carefully directives of meaning, primitive concepts, axioms and rules governing operations. In consequence it is possible to determine almost mechanically whether an expression can be regarded as meaningful and whether a proposition can be regarded as logically valid. Although the procedures involved are highly formalized, and although no attention is paid to the referents of the signs employed, the results obtained conform in a rough way to those secured by less rigorous methods. In this way logic serves to supplement and control the unanalysed operations of habitual thinking.

Chwistek devotes the major portion of the present book to the construction of a logical system which will fulfil this task. His system is not yet complete since certain portions of mathematics have not yet been incorporated within it. Neither is his system entirely adequate since parts of it are not free from ambiguity.[2] Nevertheless Chwistek's belief that a completely satisfactory apparatus can be constructed remains unshaken and even in its present state seems to him to supply methods necessary for combating anti-rationalistic philosophies.

[1] Such a system will be called a formal system.

[2] These claims will be justified in the appendix which contains an exposition and criticism of Chwistek's system of formal logic (called "semantics" or "rational metamathematics"). This appendix contains material designed to aid the reader interested in the more technical aspects of Chwistek's work. The introduction, which is addressed to the more general reader, includes only a general account of the aims and methods of the system of semantics (cf. Section II).

II

CHWISTEK'S VIEWS ON LOGIC AND THE PHILOSOPHY OF MATHEMATICS

It has been pointed out that for the most part Chwistek regards logic as a formal system [1] and it is his primary concern to develop such a formal system with the help of which it will be possible to derive various known portions of logic [2] and mathematics. For an adequate appreciation of the motives which led Chwistek to construct a *new* logical system (which he calls " semantics "),[3] it is essential to bear in mind recent developments in logic and the philosophy of mathematics.

Recent Developments in Logic and the Philosophy of Mathematics

Up to the nineteenth century mathematicians conceived their discipline as being exclusively the science of quantity. Kant, for example, regarded geometry as the study of quantitative relations of space. He claimed that the proof of geometrical propositions required a certain kind of sensuous, non-empirical, non-logical intuition (of space). Moreover he maintained that these proofs exhibit a constructive character, i.e. that they are based upon rules which stipulate the way in which the intuitions corresponding to mathematical theory must be constructed.

However with the development of projective geometry, which makes no use of metrical concepts,[4] it was soon realized that geometry might be conceived as dealing with non-quantitative relations. Additional discoveries, such as the principle of duality, geometries in which the validity of the theorems is independent of the kind of elements treated,[5] and perhaps above all non-euclidean geometries, led to the complete breakdown of the Kantian conception of geometry.

Geometry was now conceived as the study of certain abstract

[1] In this connection it should be recalled that a formal system is a system in which the directives of meaning, primitive concepts, axioms, or construction rules, and rules governing operations are precisely formulated. The theorems of such a system are derived by the application of the stipulated rules.

[2] As developed by other logicians.

[3] Or alternatively " rational semantics ", " metamathematics ", " rational metamathematics ", " formal metamathematics ".

[4] e.g. the notion of distance.

[5] e.g. the line geometry of Plücker and the sphere geometry of Huntington, in which the line and sphere respectively rather than the point were taken as the fundamental elements.

relations between unspecified (not necessarily spatial) elements. This new conception led to the application of postulational methods to geometry by such writers as Hilbert, Pasch, Veblen, Pieri, etc. Assumptions were explicitly formulated in order to make possible truly rigorous demonstrations of geometrical theorems, without any appeal to our intuition of space. Consequently the problem of the consistency of sets of geometrical axioms [1] received widespread consideration. It developed that the solution of this problem depended in turn upon the problem of the consistency of the axioms of arithmetic. In other domains of mathematics also, the attention of inquirers became directed toward providing a rigorous axiomatic foundation, with the consequence that a general study of postulational methods was inaugurated, a study which persists to this day.

During the nineteenth century foundations were also laid for the ultimate breakdown of the Kantian conception of arithmetic, as the science of quantity which depends upon sensuous intuition. The first important step in this direction was taken by Weierstrass and Kronecker, who maintained that the system of natural numbers is the basis of all branches of mathematics and that it is logically possible to arithmetize all portions of mathematics. They asserted that all mathematical entities can be defined in terms of the integers and that all mathematical results [2] can be expressed as properties of natural numbers.[3] The actual task of arithmetizing mathematics was undertaken by Cantor, Dedekind, and Weierstrass

[1] The axioms of geometry were formulated as propositional functions which contain the primitive or undefined concepts as the only variables. The only restriction imposed upon these variables is that they satisfy the axioms

At this point it may be well to recall several familiar logical distinctions. A symbol with a precisely determined meaning is called a constant. The symbol " 2 ", for example, is a constant. There are, however, symbols which have no independent meaning. Such symbols are called variables and the group of symbols, in which they occur, are called functions. For example, " *x* is a book " is a propositional function containing the variable " *x* ". If, however, this propositional function is prefixed by the phrase " for all *x* ", or the phrase " there is an *x* ", it becomes a proposition. In the function " *x* is a book ", " *x* " is called a " real variable ". In the proposition " For all *x*, *x* is a book ", " *x* " is called an " apparent variable ". A set of axioms is called consistent if it is impossible to derive any two mutually contradictory theorems from these axioms.

[2] i.e. mathematical operations and theorems.

[3] Kronecker even went so far as to advocate the elimination from mathematics as illegitimate, of all numbers other than the integers. A brief exposition and critique of Kronecker's method, which is based upon the concepts of congruence and modulus, is contained in Max Black: *The Nature of Mathematics*, New York, 1934, pp. 174–7.

among others, and led to increased rigour in the definition of the fundamental concepts of mathematics. Such notions as "limit", "area", "irrational", etc., were re-examined. New and precise definitions of these concepts were then given in terms of the integers and their relations, without any appeal to spatial or temporal intuition.

It was natural for mathematicians to suppose that if the fundamental concepts of the various portions of mathematics could be defined in terms of the integers it would be possible to unify all mathematics on the basis of elementary arithmetic. They assumed further that if they could axiomatize the various branches of mathematics, including elementary arithmetic, they could develop mathematics as a set of analytic propositions without any dependence upon intuition. The mathematicians therefore sought to axiomatize all parts of mathematics. Each part was constructed as a deductive system with its own set of primitive terms and a set of axioms concerning these primitives. The axioms were regarded as implicit definitions of these otherwise unidentified terms. This work was climaxed by the investigations of Peano, who, on the assumption that all branches of analysis had been rigidly formalized, axiomatically constructed, and reduced to elementary arithmetic, sought to complete the task of unifying mathematics by constructing an axiom system for elementary arithmetic, which would specify unambiguously the properties of the natural numbers. He constructed a system consisting of five axioms and succeeded in showing that from them it is possible to derive all the usual theorems of elementary arithmetic. His system contained three undefined concepts: "0," "number" (i.e. "integer"), and "successor". Unfortunately, however, it is possible to given an infinite number of interpretations of Peano's system which satisfy his axioms. As a matter of fact *any* serial progression whatsoever [1] satisfies these axioms.[2] It follows that Peano's axioms did not characterize the integers uniquely and that he did not supply the final basis upon which all of mathematics could be construed as a set of analytic propositions. It was

[1] The sequence of natural numbers is but one example of a progression which satisfies Peano's axioms. The sequence of even numbers is another illustration of a set of numbers satisfying these axioms. In this case "0" has its usual meaning, and the "successor" of a number is the result of adding 2 to this number.

[2] Cf. Bertrand Russell, *Introduction to Mathematical Philosophy*, New York, 1919, pp. 8–9.

thus necessary to supplement Peano's work by supplying an adequate definition of natural numbers. This task was accomplished by Frege and Russell.

Gottlob Frege surveyed the various definitions of " number ", which had been proposed by his contemporaries. After a searching critique he concluded that " number " denotes neither subjective, spatial, nor physical properties, but that although it is a non-sensible attribute, it is nevertheless an objective one.[1] He fully agreed with the general tendency of mathematicial development to construct mathematics on purely rational grounds without any appeal to psychology or intuition. His standpoint was grounded on an analysis of the different contexts in which numerical expressions occur, from which he concluded that it is possible to define the numbers in terms of certain ideas [2] so general that they belong to logic.[3] On this view it is logic as the ultimate foundation which supplies the method for unifying all of mathematics.

Independently of Frege Bertrand Russell attained essentially the same results, and it is Russell's formulation in *Principia Mathematica* which has become most widely known.[4] Using four primitive ideas [5] and ten primitive propositions, only five of which are symbolical, he developed first the principles of logic and then the various portions of analysis.[6] In other words Russell (together with Whitehead) attempted to show *in full detail* that it is possible to reduce [7] mathematics to logic.

Unfortunately, however, certain difficulties may be raised in connection with Russell's system. In the first place the question of the consistency of his system is by no means settled by his assertion that it seems impossible to doubt or

[1] Cf. Bertrand Russell, *Our Knowledge of the External World*, New York, 1929, p. 218.

[2] Examples of such ideas are " implication " (i.e. " if . . . then "), " negation " (i.e. " not "), etc.

[3] i.e. he defined " number " as a property of a property of a collection. For present purposes it is sufficient to consider a collection as a set of objects.

[4] Russell's system cannot be called a formal system in the sense in which we have been using this term, since he did not stipulate all the rules of procedure to be used in deriving theorems.

[5] i.e. " elementary proposition ", " negation ", " assertion ", and " disjunction ".

[6] Amongst other things Russell was able to prove Peano's five postulates.

[7] In this context the process of reduction requires the definition of the concepts of mathematics in terms of logical concepts, the statement of the axioms and theorems of mathematics in terms of logical concepts, and the proof of these axioms and theorems by purely logical devices from the axioms of logic.

deny any of the principles of logic. As Hilbert [1] already pointed out, the problem of the consistency of a set of axioms requires consideration in its own right and Russell himself never gave any serious thought to this problem. Furthermore it is difficult to reconcile Russell's assertion on the one hand that logic is concerned with the real world,[2] and his insistence on the other hand that the laws of logic are true in all possible worlds [3] (i.e. that logical principles are relevant to a realm of entities which are not necessarily existent). Moreover other difficulties arising in connection with the platonic realism expressed by the latter point of view prevented Russell from offering any consistent view concerning the nature of classes.[4]

But for our purposes the difficulties which arise in connection with the systematic development of the theory of classes are most important. This theory, which serves as the foundation of modern mathematics, had been extended by Georg Cantor to include infinite (transfinite) classes as well as the usual finite ones. Unfortunately, however, it was soon shown that it is possible to develop a number of paradoxes within this theory. Typical of such paradoxes is the contradiction of Burali-Forti (151), the first paradox to be demonstrated within the theory. In order to understand some of the suggestions made to eliminate such obvious violations of the principle of contradiction, it is worth while formulating two of the simpler paradoxes, that of Russell concerning classes which are not members of themselves, and the paradox of the liar, sometimes called the Epimenipes paradox. Russell's paradox is as follows : If a class is conceived as a set of objects it is possible to form a class of such classes. It then seems to follow that certain classes include themselves as members. Thus, if non-men form

[1] Cf. p. xxx.

[2] Russell, *Introduction to Mathematical Philosophy, l.c.*, p. 169.

[3] *ib.* p. 192.

[4] Russell originally regarded classes as mere aggregates of terms or things. On this view the null class became a meaningless concept. In addition he soon realized that classes must have a different kind of reality than things, since in the course of his investigations he found it necessary to distinguish between a term and the class whose only member is that term. Consequently he abandoned his original conception of classes and advanced what he called the " no-class " theory. Since for practical purposes and under certain conditions functions of a function of a variable can be regarded as functions of the class determined by that variable, he maintained that it is possible to develop the theory of classes without ever using the concept of a class itself. On this view classes are but logical fictions, i.e. symbolic or linguistic shorthand devices.

a class, this class appears to be a member of itself since it is not a man (150–1). Bertrand Russell raised the question whether the class of all classes, which are not members of themselves, is a member of itself. Two contradictory answers can be given to this question. Where the symbol **A** is used to denote the class of all classes which are not members of themselves,

> if **A** is a member of itself, by definition it is not a member of itself;
> if, however, **A** is not a member of itself, it is a class which is not a member of itself, and consequently is a member of itself.

The paradox of the liar can be formulated as follows: When I say that I am lying,

> if I am lying I am telling the truth;
> if, however, I am telling the truth then I am lying.

Russell proposed to resolve these paradoxes by distinguishing three different kinds of statements: true, false, and meaningless.[1] He regards statements as meaningless when they fail to conform to a certain set of rules, which he calls the theory of logical types. These rules formulate the permissible ways of combining logical ideas. When Russell suggested such a set of rules he developed what has come to be known as the simple theory of types. In this theory a distinction is drawn between individuals,[2] functions which take individuals as arguments[3] (i.e. functions of type **1**), functions which take functions of type **1** as arguments (i.e. functions of type **2**), etc. In other words the type of a function is determined by its argument.[4] A class can be a member only of classes, not of any class whatsoever. Hence it is meaningless to speak of a class being a member of itself. Thus the statement in Russell's paradox are neither true nor false but meaningless statements, and it is impossible for the paradox to arise in significant discourse. It turns out, however, that while it is possible to resolve paradoxes such as Russell's with the help of the simple

[1] In this context "statement" is not to be identified with "sentence" or "proposition". "Meaningless" does not indicate a third truth-value.

[2] i.e. any object which is neither a function nor a proposition. Individuals are of type **0**.

[3] For example, in the function "*x* is a man", "*x*" is the argument to the function, which takes individual arguments only; in the function "*R* is transitive", the argument "*R*" will have only functions as values.

[4] Each logical function belongs to a single logical type. Moreover its arguments must be of the immediately preceding type.

theory of types, such antinomies as the Epimenides paradox cannot be eliminated by it.[1]

For this reason Russell proposed the branched or ramified theory of logical types. In this theory the type of a function is determined not only by the type of the arguments which it takes, but also by the form of the function. The theory is stated in terms of the notion of the " order " of a propositional function or of a proposition. A predicative function of an individual or a first-order matrix[2] is defined as an elementary[3] function of an individual. First-order functions are defined as functions whose arguments are individuals or are obtained from such functions by quantification.[4] A second-order matrix is a function which involves at least one first-order matrix among its arguments but has no arguments other than first-order matrices and individuals. Second-order functions are defined as second-order matrices or functions obtained from the latter by quantifying some of the variables, and so on for functions of higher order. An analogous hierarchy of propositions can easily be specified. It turns out that the branched theory of types is sufficient to remove all of the paradoxes which have been developed in the theory of classes.

Chwistek's Early Contributions to Logic and the Philosophy of Mathematics

It is in connection with the theory of logical types that Chwistek made his earliest contributions to logic and the philosophy of mathematics. His achievements were two-fold. In the first place he was the earliest logician to advocate anew the simple theory of types for the elimination of the

[1] As Ramsey pointed out Russell's paradox involves only logical concepts, while the paradox of the liar is based upon a non-logical concept, the concept of " truth " as well as logical concepts. Cf. Ramsey, *The Foundations of Mathematics*, New York, 1931, pp. 20–1.

[2] While in this introduction Russell's own statement of his views is followed, it is worth noting that " matrix " is a syntactical term (cf. p. xxxvi, n. 1) while " function " and " individual " are not. Strictly speaking therefore, predicative functions should not be identified with first-order matrices.

[3] An elementary function is a function which contains no quantifiers, i.e. a function which contains neither the universal operator " for all (every) x " nor the existential operator " for some x ", or " there exists at least one x ".

[4] Quantification is the process of asserting a propositional function of all or some values of one or more variables. If all the variables are quantified the function becomes a proposition. If only some of the variables are quantified the function remains a function.

paradoxes of the theory of classes.[1] And secondly he seriously criticized the use of existence axioms in logic and mathematics.

As it happens the ramified theory of types does not permit the development of Cantor's theory without an additional assumption, known as the Axiom of Reducibility. This axiom asserts that every propositional function[2] of any order whatsoever is formally equivalent to some propositional function of order one.[3, 4] In his earliest consideration of Russell's work[5]

[1] It is perhaps worth while to point out that several other suggestions have been made for avoiding these antinomies. Zermelo and Fraenkel, for example, maintain that the paradoxes need not occur if the axioms of the theory of classes are carefully formulated.

Other writers seek to eliminate the paradoxes by means of a distinction between an object language and its various " metalanguages ". An object language is a language which is the object of investigation. " Metalanguages " are of two kinds : syntactical and " semantical ". The syntactical language is the language in which the forms of the sentences of the object language are studied. The " semantical " language is the language in which the relations between the symbol and the thing symbolized are investigated. For example, " It is snowing " is a sentence in the English language, which can be regarded here as an object language. The sentence " ' It is snowing ' contains three words " is a syntactical statement since it is concerned with the structure of one of the sentences of the object language and states a syntactical property of this sentence. The sentence " ' It is snowing ' is true if and only if it is snowing " is a " semantical " statement stating a " semantical " property of this same sentence of the object language. Hilbert's distinction between a language and its " metalanguage " corresponds to the distinction drawn here between an object language and its syntax language. Professor Alfred Tarski initiated formal investigations in " semantics ". Cf. " O pojęciu prawdy w odniesieniu do sformalizowanych nauk dedukcyjnych " (" On the Concept of Truth in reference to Formalized Deductive Sciences "), *Ruch filozoficzny*, vol. 12, 1930–1, pp. 210–11, and " Der Wahrheitsbegriff in den formalisierten Sprachen ", *Studia philosophica*, vol. 1, 1936, pp. 261–405, a translation of a work which appeared originally in Polish. Both the syntax language and the " semantical " language can be constructed as formal systems and a theory of types can be specified for each of these languages. Many writers, on the basis of Ramsey's distinction between the logical and non-logical (" semantical ") paradoxes (xxxv, n. 1) make use of the (simple) syntactical theory of types for the elimination of the logical paradoxes and the " semantical " theory of types for the resolution of the non-logical paradoxes.

The above usages of the terms " metalanguage " and " semantics " must be distinguished from Chwistek's use of these terms, which will be considered below (xxxviii ff. and Appendix). It is for this reason that these terms have been inserted in quotation marks in the present discussion.

Chwistek has never specifically commented upon any of these methods of avoiding the paradoxes. However, his general attitude toward existence axioms and toward the distinction between a language and its " metalanguage " (xli) are sufficient to indicate that he would emphatically reject any of these proposals.

[2] Of individuals as arguments.

[3] This assumption, which largely removes the distinctions drawn between the orders within the types, is necessary in the development of the theory of real numbers. Two functions are said to be formally equivalent if they are (materially) equivalent for all values of the variables contained in these functions. Cf. p. xlvii, n. 1.

[4] And similarly for higher types.

[5] *Z.S.*

an analysis of *Principia Mathematica*, Chwistek seriously questioned this axiom.[1] He suggested that the situation be remedied by retaining the branched theory of types but rejecting the axiom in question. This proposal involves the identification of classes with propositional functions and consequently the development of a modified theory of classes.[2] A number of years passed before Chwistek worked out in full detail the radical modifications required in developing a theory of classes subject to this far-reaching restriction.[3] He then pointed out the real difficulty in connection with the Axiom of Reducibility : it is not a proposition of logic but an existence axiom, with whose help it is possible to " prove that there are objects which perhaps cannot be determined " even though " to have any object it is necessary and sufficient to have a proposition from which this object is to be obtained by a wholly determined formal process ".[4] In the interim between Chwistek's proposal of the theory of constructive types and its actual construction, he suggested a return to the simple theory of types.[5] Unfortunately, however, this theory also depends upon existence axioms, e.g. the Axiom of Infinity, which asserts the existence of infinitely many individuals.[6]

The Meaning of Semantics

Thus while Chwistek clearly indicated his dissatisfaction with Russell's attempt to complete the refutation of the Kantian thesis concerning mathematics, he was also aware of the shortcomings of his own early views on the theory of types.

[1] As a matter of fact the grounds which Chwistek gave for his criticism of this axiom turned out to be utterly false, as he himself later realized. He asserted that the Axiom of Reducibility led to a contradiction within Russell's system. What Chwistek actually demonstrated was that when a certain postulate of Poincaré is added to the axioms of Russell, a contradiction ensues. This postulate was formulated by Poincaré as follows : " Consider only objects which can be defined in a finite number of words." Chwistek has always accepted this postulate together with its implications.

[2] In this theory identity holds only between classes.

[3] Cf. *T.C.T.* Chwistek called the new system which he developed the theory of constructive types, or the pure theory of types. It was characterized by the fact that in conformity with Poincaré's postulate it contains only a finite number of primitive symbols, and a long series of verbal directions for the construction of additional symbols and for the transformation of expressions in a *finite* number of operations. Its proofs are completely symbolic.

[4] *T.C.T.*, p. 10.

[5] *A.L.F.*

[6] Chwistek pointed out that alternative existence axioms can be assumed and that each choice of axioms leads to a distinct theory of classes.

Consequently he soon proposed a new formal system which he called semantics,[1] which he hoped would accomplish what Russell had failed to achieve. He wished to include in this new system those features of previous systems which he considered valid, and at the same time to avoid the difficulties encountered in these systems.

Chwistek felt, for example, that Bertrand Russell was on the right track when he attempted to reduce mathematics to logic. Yet in so far as the theory of logical types presented by Russell as a necessary concomitant of his system was not entirely satisfactory, Chwistek did not feel that Russell had achieved the end he had in mind, i.e. the unification of mathematics. Nevertheless he wished to retain not only Russell's general aim but the deductive method of presentation which Russell employed in *Principia Mathematica.*

Actually he went beyond Russell when he began to develop the latter's suggestions. In the first place even though Russell failed to derive mathematics from axioms which undoubtedly belong to logic, Chwistek still maintained that it is possible to unify mathematics, if not with the help of logic alone, then with the help of semantics.[2] In other words the system of semantics is the result of an extension of the logistic thesis; Chwistek asserts that mathematics *and* logic can be reduced to semantics.[3] In the second place he not only developed the

[1] The determination of the meaning of this term is the problem under investigation in the next few pages. Nevertheless it may now be said that whatever meaning Chwistek assigns to this term his usage must be distinguished from that of most contemporary writers on the subject. He does not intend to develop a theory of meaning, nor to enter into contextual analysis. In spite of his attack upon the hypostatization of entities (xxviii), he is obviously not seeking the referents of words. Moreover he is not concerned with the study of the responses of individuals to the names of entities. He does not analyse the relations between the symbol and the thing symbolized, i.e. such concepts as " truth ", " designation ", " satisfaction ", etc. This, of course, is not to say that he has no interest in any of these investigations. It is just that these projects are not the concern of semantics in his sense of this term. On his view semantics is concerned neither with psychological nor non-formal investigations. Chwistek explicitly rejects a distinction between a symbol and that which it symbolizes.

[2] i.e. with the help of logical and semantical concepts.

[3] Evidence for this assertion can be found *only* in Chapter VII, where Chwistek's system is worked out rigorously. Chapters IV–VI should be considered as an attempt to familiarize the reader with Chwistek's technique and to supply the motivation for its introduction. Although a number of clues which turn out to be very helpful in understanding the later chapter are given in these earlier chapters, the proofs contained in the latter are intuitive. The gap between logical and semantical notions is not completely bridged and Chwistek's theory cannot be evaluated on the basis of this material.

system of semantics deductively but as a formal system. Thus the system of semantics is an attempt to define the concepts of logic and mathematics in terms of two primitive semantical concepts " * " and " c ", where all the construction and transformation rules governing semantical concepts are explicitly formulated.[1] The axioms and theorems of mathematics and logic are stated in terms of the primitive and defined concepts of semantics, and the axioms and theorems of these domains are proved with the help of the transformation rules of semantics. These proofs are purely symbolic.[2]

Henri Poincaré suggested [3] several rules for the conduct of logical investigations concerning the infinite. He advised [4]:

1. " Consider only objects which are capable of being defined in a finite number of words."

2. " Never lose sight of the fact that every proposition which concerns the infinite is a translation, an abbreviated statement of propositions which refer to the finite."

3. " Avoid classifications and definitions which are not predicative."

Well aware of the situations in which Poincaré's rules are of value, Chwistek adopts them in slightly modified form, formulating them in such a way that they can be applied to the concepts of semantics. It is characteristic of Chwistek's procedure that none of these rules appear either among the

[1] The sign " c " is called an expression (xl) as is any combination of the two primitive signs which is obtained with the help of these carefully stipulated construction rules.

[2] For further details concerning the technical aspects of Chwistek's system see the Appendix.

[3] Poincaré's suggestions were intended to achieve two objectives: first, the avoidance of the paradoxes of the theory of classes; second, the development of a general method for constructing mathematical entities, which will possess certain properties demonstrated to exist. The theorem which states that there is no greatest prime number is an example of this second point. It can be shown that this theorem is true, for if any prime number p is taken to be the greatest possible prime, and the product $1.2.3.5. \dots .p$ formed from all previous primes, a new number p' can obviously be constructed by adding **1** to this product. This number, if it is not itself prime, is divisible by a prime which must be greater than p. It is possible to determine in a finite number of steps whether p' is prime. If it is not prime, it is possible to determine in a finite number of steps by which prime it is divisible. This proof does not actually require that this new number be calculated. As a matter of fact it would be impossible for any individual to carry out this proof for every possible case. Even for cases where p is relatively small and only a few steps are required to calculate the new number p', it would be very impractical to compute its value.

[4] Henri Poincaré, *Dernières Pensées*, Paris, 1913, pp. 138–9.

rules governing the system of semantics or in the system of semantics itself. He regards the first rule, which is an instrument for the criticism of classical Mengenlehre, as a special case of Occam's razor,[1] one of the criteria of sound reason. In as much as the entire system of semantics is constructed in conformity with the method of sound reason he finds no need to include this postulate among the rules of his system. As a rule governing the system of semantics, it obviously cannot be contained within this system.

Nevertheless Chwistek minimizes the negative role of this postulate, since he feels that it requires the modification and reconstruction of classical mathematics rather than the rejection of large portions of this subject. For this reason he gives a positive interpretation of Poincaré's first rule, an interpretation which is based upon the ambiguity of the word " word ".[2] He formulates this postulate as follows : " Consider only objects which are capable of being defined in terms of a finite number of expressions." Although this interpretation of Poincaré's principle does not prevent the construction of a formal system which would include mathematics,[3] it does impose definite restrictions upon any such construction. In the first place the concept " expression " must be a basic concept of any such system.[4] In the second place fundamental revisions are required in the classical theory of classes, particularly with regard to such infinite classes as the real numbers. Poincaré's second rule, which must also be interpreted as a restriction placed upon the construction of expressions,[5] is important in this connection, since it is applied in conjunction with the first postulate. Thus all expressions even those concerning infinite classes can be constructed with the help of a finite number of expressions.[6]

Chwistek's thesis that a theory of types [7] is essential for

[1] Cf. *T.L.*, pp. 125–6.

[2] This ambiguity exists in French and Polish as well as English.

[3] As Poincaré himself would maintain.

[4] Thus " expression " is a technical term of Chwistek's system of semantics and is therefore placed in quotation marks. Since in subsequent discussion this term will be used only in the sense indicated and since Chwistek himself does not employ quotation marks, they will be omitted in what follows.

[5] Since propositions are defined in terms of expressions.

[6] Expressions which can be constructed in a finite number of steps by the application of stipulated rules will be called constructible. This is not to say that these expressions must actually be constructed. It is essential only that it is possible *in theory* to construct them.

[7] Either the simple *or* pure theory of types.

the elimination of contradictions and meaningless statements may be regarded as a reformulation of Poincaré's third rule. Like the other postulates, and for the same reason, such a theory is not found among the explicit rules governing the expressions of Chwistek's system. Nevertheless, every statement of this system is constructed in conformity with such a theory.

In his investigations on postulational methods Hilbert found it necessary to study the structural properties of signs. Chwistek was apparently much impressed by this aspect of Hilbert's work. He points out that the great merit of the formalist school,[1] lies in its initiation of syntactical investigations concerning the properties of systems of symbolic logic. But he seems not to recognize that for Hilbert syntactical investigations constitute a field of study distinct from logical investigations themselves. Indeed, Chwistek asserted :

> " . . . Professor Hilbert assumes a system of axioms containing the principles of the Logical Calculus together with some purely Mathematical axioms (e.g. Zermelo's axiom) ; and he endeavours to prove with the help of " metamathematical " methods that they imply no contradiction. . . . Suppose he has proved by means of these primitive ideas and propositions " [of the logical calculus] " that a system of propositions (say *p*, *q*, *r*) is compatible with them. Then he has simply proved these propositions. If he has used (explicitly or tacitly) other ideas or propositions, then he has assumed some new hypotheses, which appear more general than Zermelo's axiom, etc. . . . Note that Hilbert does not assume the Theory of Types . . . such a ' metamathematic ' cannot be essentially different from the Logical Calculus, this calculus being as a matter of fact a simple consequence of the laws of our thinking." [2]

In a footnote to this same paragraph Chwistek adds :

> " . . . there is a Meta-mathematic dealing only with the meaning of symbols, but never with the truth or falsehood of propositions. Therefore there is no means of providing a mathematical or logical proposition with such a Meta-mathematic." [2]

It would therefore seem that Hilbert's influence upon Chwistek's intellectual development has been more apparent

[1] Of which Hilbert was the founder.

[2] *T.C.T.*, p. 11. Spelling, typography, the usage of capitals and quotation marks conform to the original text.

than real except in regard to terminology.[1] In fact Chwistek's own procedure when he develops the system of semantics is the very antithesis of that suggested by Hilbert. He does not follow Hilbert in distinguishing between mathematics and " metamathematics " ; thus, he does not construct one language whose concepts form the basis for the derivation of logic and mathematics, and a second language whose theorems state the syntactical properties of the first language. Rather he constructs a single language, which includes theorems concerning the structural properties of expressions in addition to logical and mathematical theorems.

We are now in a position to formulate more clearly what is meant by semantics. Semantics is a formal system which makes use of a carefully specified symbolism based upon two signs " * " and " **c** ".[2] The sign " **c** " is called an expression. Any combination of these signs obtained by the application of carefully stipulated rules is also called an expression. It is evident that the expressions of semantics must be constructible in the sense in which we have defined this term. The symbols usually employed in logic and mathematics are correlated with certain of these expressions and are allowed to replace them.[3] Chwistek himself defines semantics as the " science of expressions ". In other words semantics treats only those configurations of signs which are expressions in the sense indicated. Further rules are stipulated whose application to expressions yield new configurations called theorems. These theorems are of two kinds. Some of them are sentences of the language of logic and mathematics. The remaining theorems formulate syntactical properties of this object language. Nevertheless both kinds of theorems are derived by a single method.

It is clear then that Chwistek has attempted to achieve within a single system three distinct objectives, which may be briefly indicated by the rubrics " constructibility ", " metamathematics ", and " reducibility to a more general basic science ". These elements are borrowed from the representatives

[1] i.e. Chwistek borrowed the term " metamathematics " from Hilbert. It should be noted that Chwistek's usage of this term in the passage cited differs from his usage in the current work. But each of Chwistek's usages differs from that of Hilbert.

[2] In Chwistek's eyes one of the lessons learned from the paradoxes is that they are caused by the ambiguities of the words used in everyday language. Consequently the various ideas evoked by words must be carefully distinguished and each idea represented by a specific sign.

[3] In practice these symbols are regarded as expressions.

of the three schools of thought engaged in the discussion of the philosophy of mathematics during the present century: the intuitionists or constructivists, the formalists, and the logisticians.[1]

III

THE THEORY OF PLURAL REALITY

Chwistek has been concerned with the problem of reality for many years and first stated his views on the subject almost twenty-five years ago in a lecture delivered before the Philosophical Society in Cracow.[2] A few years later the material presented at that time was published in considerably amplified and modified form.[3] Chwistek's views became the subject of heated debate in Poland, although the severe criticism to which they were subjected[4] was based on interpreting Chwistek's formulations without reference to the context in which they occurred.

Chwistek never intended his views concerning the problem of reality to constitute a new metaphysical theory. On the contrary he specifically stated that " the problem of the present study is the establishment of the meaning of the term ' reality ' ".[5] His theory of plural reality must therefore be regarded as an attempt to specify the various ways in which the term " real " is used[6] and not as an attempt to provide a solution of the " philosophical " problem of reality. Accordingly it is a misinterpretation of Chwistek's intent to take literally his assertion that there is no one true reality but that

[1] This is not to say that Chwistek accepts all the ideas advanced by the writers in question. For example, he rejects Russell's theories of types because they depend upon existence axioms. He rejects Poincaré's opposition to constructions based upon the use of logical symbolism, and his stress upon the necessity of mathematical intuition (80).

[2] Cf. *T.L.*, pp. 139–151.

[3] Cf. *W.R.*

[4] Cf. e.g. *Przegląd Warszawski*, Rok 2, Tom 1, 1922, which contains a review of *W.R.* by Professor Tadeusz Kotarbiński, pp. 426–8, and an article by K. Irzykowski, pp. 291–306.

Przegląd filozoficzny, vol. 25, 1922, which contains a critique of Chwistek's views by Dr. Roman Ingarden, pp. 451–468.

Determinism nauk przyrodniczych (*The Determinism of the Natural Sciences*), by Joachim Metallman, Kraków, 1934, pp. 52–7.

[5] Cf. *W.R.*, p. 3.

[6] In conformity with this interpretation the phrase " concept of reality " has been used throughout Chapter X to render the word " rzeczywistość ".

there are at least four different realities. Moreover, it is not a relevant criticism to point out that on his analysis it is impossible to account for all aspects of experience. Chwistek himself is not interested in such questions as whether any of the well-known cosmological theories is the only theory which can account for the totality of experience. He notes, for example, that not only do both materialism and idealism seem valid to their adherents, but that to one and the same individual materialism may at one time appear to be the only valid doctrine, while at another time idealism alone may seem to explain certain portions of experience in a satisfactory manner.

This interpretation of Chwistek's theory of reality as an attempt to distinguish the different usages of the word " real " conforms to his general nominalistic approach (xxvi–xxvii). On his view the proper task of philosophy is definition rather than demonstration.[1] Accordingly, he regards the enumeration of the various ideas evoked by the term " real " [2] as genuine philosophical activity, whose object is the attainment of maximum certainty in knowledge within the limits of human reason. Sound reason, when applied to the problem of reality, therefore, prevents ambiguities and the confusions arising from them.

However in *The Limits of Science* Chwistek does not actually define the four meanings of the term " real " which he distinguishes ; and he speaks of the " criteria " of the various realities without specifying them. What he does is to suggest four different contexts in which the word " real " is used. Thus he notes that " real " is a predicate employed in connection with four different kinds of entities : atoms, things and persons, images,[3] and sensations. Accordingly, he distinguishes four different " concepts of reality ", that of physical reality, of natural reality, of the reality of images, and of the reality of sensations. In so far as each of the entities mentioned is characterized by a different set of properties, a different " concept of reality " is employed each time one of them is called " real ". Although Chwistek himself does not regard this classification of the various " concepts of reality " as

[1] Cf. *W.R.*, p. 7.

[2] Chwistek asserts the importance of distinguishing the various meanings of a term because the use of the same term in different meanings, i.e. " operating with different patterns of reality " (270), is unavoidable in everyday language and because there are no criteria on whose basis it is possible to decide which meaning should be employed at a particular moment.

[3] e.g. dreams.

exhaustive it seems worth while to point out some of the more obvious omissions in his account. He does not, for example, make clear in what sense macroscopic objects are said to be " real ". Neither does he state whether such entities as genes and chromosones are to be regarded as " real " in the same sense as are atoms. Nor does he specify in what sense certain numbers can be regarded as " real ".

Nevertheless in his earlier writings on the theory of plural reality Chwistek did attempt to define the various meanings of the word " real ". At that time he not only regarded *Principia Mathematica* as the model for all deductive systems, but he maintained that various portions of philosophy, in particular those portions which deal with reality, can be formulated as deductive systems. He called such formulations " formalizations ", and maintained that their primitive concepts and axioms are not arbitrarily posited but are derived from an analysis of experience. These axioms contain concepts specific to the theory of reality as well as certain logical concepts. In stating these axioms,[1] ten in all, Chwistek employed six propositional functions as primitive.[2] He contended that certain sets of axioms chosen from this group implicitly define the different meanings of the word " real ".[3] He suggests

[1] Cf. *W.R.*, pp. 30–4.

(1) If an object is given immediately it is real.

(2) If an object is visible, it is real.

(3) If an object is real, it is visible or given immediately.

(4) Certain real objects are not visible and are not given immediately

(5) An object is visible if and only if it is visible during waking life.

(6) There are objects which are visible, which need not be visible during waking life.

(7) An object is visible if and only if it is visible under normal conditions.

(8) There are objects which are visible, but which need not be visible under normal conditions.

(9) Part of a real object is real.

(10) If part of an object is real, that object is real.

This set of axioms is a contradictory set.

[2] " x is real," " x is given immediately," " x is visible," " x is visible during waking life," " x is visible under normal conditions," and " x is part of y " (cf. *W.R.*, *l.c.*).

[3] Cf. *W.R.*, p. 33.

Sensational reality is defined by axioms 1, 2, 3, 5, 8.

The reality of images is defined by axioms 1, 2, 3, 6, 8.

Natural reality is defined by axioms 1, 2, 4, 5, 7, 9, 10.

Physical reality is defined by axioms 1, 2, 4, 5, 8, 9, 10.

The adequacy of Chwistek's definitions is not in question here although some of the problems raised by his analysis have already been indicated.

further that from each of these sub-sets important theorems concerning " reality " may be derived, although he himself did not derive any of them.[1]

The fact that in *The Limits of Science* Chwistek does not construct such formalizations might lead one to suspect that he has abandoned this method. However, in a recent letter (28th May, 1939) he states: " I have not abandoned this conception, although I think it has only theoretical importance." And indeed, in the book itself, Chwistek makes sufficiently clear that the formalization of reality is possible, though it requires to be based on semantical considerations. He points out that it is possible to construct symbolic representatives of the objects of experience, i.e. configurations of signs denoting these objects (268). He also contends that it makes no difference whether signs are interpreted as things, collections of atoms, visions,[2] or expressions (85). He suggests that it is possible to correlate with signs not only logical and mathematical concepts but philosophical concepts as well.

Chwistek's position on questions of logical theory have influenced the formulation of his views on the problem of reality. He requires, for example, the acceptance of a theory of types prior to the formalization of reality. With the help of this theory he distinguishes an infinite number of meanings of the word " real " in addition to the four meanings already indicated. For there are formalizations of higher type which take formalizations of lower type as arguments. However, this theory of types, which Chwistek calls " metascientific ", is not formulated very precisely. Thus while Chwistek maintains that each of the four formalizations (i.e. each of the four " concepts of reality "), are of a different order although they

It should be noted that different concepts of reality are based on contradictory axioms. The concept of natural reality for example is based on axiom 7, which is the contradictory of axiom 8, upon which the concept of physical reality is based.

Chwistek maintains that the four " concepts of reality " thus defined are equally plausible and that it is possible to develop a consistent philosophical doctrine on the basis of each of these concepts. Consequently he asserts that no one philosophical theory is to be preferred to any other. Nevertheless Chwistek feels that a philosophical view based upon the concept of the reality of sensations conforms more adequately to the nominalistic approach than any other philosophical position.

[1] None of these deductive systems can be regarded as formal systems since their formation and transformation rules have not been explicitly formulated. Nevertheless they might be developed as formal systems.

[2] Visions are one kind of images.

are of the same type,[1] he nowhere sets up a precise hierarchy of orders. Nevertheless at very isolated points he does venture to make such comments as: the concept of physical reality is of higher order than the concept of natural reality (279). In spite of the lack of an explicit formulation of the " metascientific " theory of types this theory is of use in resolving some of the epistemological puzzles raised in connection with dreams. Chwistek maintains, for example, that it is not an error for an individual to regard his dreams as " real " Dreams are just as " real " as are persons or things. They are merely of a different order. On the other hand it would be wrong for an individual to regard the sensation which he experiences when he is dreaming as sensations of the same type as those which he experiences when he is awake. Chwistek's contribution to philosophical theory thus rests on the method he has devised by which it is possible to obtain precision of philosophical concepts.[2]

[1] Bertrand Russell's formulation of the branched theory of types is based upon a distinction between various orders of functions and propositions. First-order functions, for example, are defined as functions whose arguments are individuals. Although all such functions are not of the same type Russell pointed out that in practice these differences of type are neglected. (Cf. Whitehead and Russell, *Principia Mathematica*, vol. i, Cambridge, 1935, pp. 161–2.) Chwistek has therefore interpreted Russell as maintaining that although functions are of different orders, they are all of the same type (cf. *Z.S.*, p. 320). Consequently when Chwistek introduces a " metascientific " theory of types in connection with the theory of plural reality, he maintains that functions of different orders may be of the same type.

[2] In his considerations of the problem of reality Chwistek has on occasion alluded to general semantics (as distinct from rational semantics) and seems to suggest its importance in dealing with the problem of reality. He does not, however, specify exactly what he understands by the term " general semantics " and always returns to rational semantics, the system of semantics developed at length in this book, for hints to be applied in resolving this problem.

Typical of this procedure is Chwistek's insistence that it is possible to treat only " patterns of reality ". For he never explicitly indicates what is to be understood by this term. Obviously the word " pattern " is not used in precisely the same way as it was employed in Chwistek's logical considerations (296, n. 2), since he maintains that it is impossible to obtain singular propositions such as " a is real " [where " a " denotes a particular object (268–9)], with the help of rational metamathematics or semantics. Nevertheless it is clear that a " pattern of reality " is some kind of a function since it contains a variable. The pattern " x is real ", for example, contains the variable " x ".

IV

The Philosophy of Science

Chwistek's interest in the problems of the philosophy of science, as manifested in his published writings, is comparatively recent. As a matter of fact his work in this field has been confined almost entirely to a critical account of some of the fundamental concepts of the natural sciences.[1] Semantical considerations and the theory of plural reality, as theories constructed in accordance with the method of sound reason have therefore had an important role in the development of, Chwistek's views on the philosophy of science.

Chwistek's Conception of Science

Chwistek views science as an attempt to develop a consistent rationalistic view of the world, based upon simple, clear " truths " derived from experience by the application of sound reason (3). He points out three distinct elements involved in scientific activity: classification, description, and explanation. Although none of them is sufficient to characterize the method of science completely, scientists do nevertheless classify, describe, and explain phenomena (3). Explanation is given in terms of laws usually numerical in character (25). For this reason scientists must specify a conceptual apparatus which will permit the simplest possible solution of concrete scientific problems. Accordingly economy is not an end in itself but is relative to the particular problems in which the scientist is interested.

[1] Chwistek does include positive constructive work on the special theory of relativity but his views on this subject will not be considered here. In the first place since his early consideration of relativity theory (*G.N.*, pp. 215–222), his position has undergone continual evolution (cf. *L.R.* and *P.P.R.S.*), and there is no way of knowing whether these views have attained their final formulation. In the second place the present formulation of Chwistek's views on this subject (242–252), which is obviously quite different from his early position, was originally written in English. Unfortunately certain passages are not entirely clear (cf. e.g. p. 245). Present world conditions have made it impossible to clear up these obscurities. Furthermore this introduction is concerned with material of philosophical rather than of purely scientific interest. It will therefore be sufficient to point out that in connection with the theory of relativity Chwistek is interested in deriving the Galilean transformation, which implies that it is impossible to detect by means of mechanical experiments uniform rectilinear motion with respect to absolute space, and the Lorentz transformation, which implies that two events, which are simultaneous for an observer at rest in a given frame of reference, no longer appear so to an observer moving relative to that frame of reference.

Chwistek points out that in spite of the fact that the scientist endeavours to describe and explain phenomena, he is not interested in finding their " causes ". Such questions as: " Why does the earth revolve around the sun ? ", " What is an atom ? ", etc., are metaphysical rather than scientific; they lead to fruitless investigations, which do not extend the scope of our experience (11). The extension of our experience is one of the primary aims of the scientist and involves the possibility of prediction on the basis of scientific laws.

Chwistek also realizes that our scientific knowledge is not all-inclusive. Nature does make sudden jumps (51). It is frequently possible to give explanations of phenomena in terms of scientific laws only *after* their occurrence. It is therefore impossible in many situations to make predictions of a kind which would prove useful. For this reason scientific knowledge in particular, as well as knowledge in general, can never be complete (xxvii).

Finally in conformity with his insistence that science must be based upon the method of sound reason, Chwistek opposes the introduction of " metaphysical ", " ideal ", anthropomorphic, and " fictional " concepts into science (xxiv, n. 1). He also objects to the use of rough analogies, because the scientist is likely to forget that they are of value only as auxiliary devices and cannot be regarded as accurate representations of scientific facts.

Such is Chwistek's general conception of science, a conception which he works out with the help of a critical analysis of material drawn from the different sciences. However when he turns to particular problems of the philosophy of science Chwistek selects for consideration material taken from the natural sciences rather than from the biological or social disciplines. As a matter of fact he discusses the methodological problems of physics almost exclusively.

Measurement and Arithmetic

Chwistek points out that in physics events are abstracted from the totality of experience with the help of sets of numbers. Each such set is called the spatial representation of an event. This process of abstraction, i.e. of " formalizing reality " [1]

[1] This meaning of the term " formalizing reality " obviously differs from that employed elsewhere (xlv), in so far as it involves a *mathematical* representation of reality (238).

is a device used to predict (238). The apparatus employed in this process evokes images of reality, but it must not be confused with actual events. It is for this reason that Chwistek sharply criticizes those who take these " images " literally (239), i.e. those who identify " concepts of reality " with " reality ". On Chwistek's view mathematics supplies an adequate apparatus for representing the properties of subject-matter without requiring any " metaphysical " assumptions. The derivative " $\frac{dx}{dy}$ ", for example, which is employed by the physicist is not a mathematical fiction as some writers maintain. For the velocity, which is represented by " $\frac{x_2 - x_1}{t_2 - t_1}$ ", would in that case also have to be regarded as a mathematical fiction,[1] a conclusion few physicists would be willing to accept. On the other hand Chwistek regards both the concept " derivative " and the concept " velocity " as expressions. Since the physicist makes discoveries which " transform the surface of the earth " (69) with the help of simple operations upon velocities, etc., Chwistek feels that it is vital for the physicist to know how to use these expressions, i.e. to perform these mathematical operations upon them.

Chwistek treats as expressions even the numbers used in the process of measurement. The theorems of arithmetic are derived with the help of logical and semantical devices alone and Chwistek insists that they need *not* be verified by reference to experience. The actual process of measurement he regards as a " crudely defined activity " (255), since it is impossible to set up a one to one correspondence between the results of measurement and real numbers. A physicist interested in measuring the length of a table would not be satisfied with a single measurement but would make several. He would then formulate the results of these measurements as a series [2] of numbers.[3] He would say that the length of the table is represented by a number greater than the smallest number

[1] It should be recalled that for Chwistek the word " fiction " is a term of opprobrium. He has already indicated that the concepts of the calculus are not fictions, since they can be developed in terms of the concepts of semantics. He suggests that they be introduced in a purely formal way in such a manner that they involve only constructible expressions (321–3). He has indicated that the concept of velocity at a point can be analysed in terms of the concepts of the calculus (200–3).

[2] In this context the word " series " is used in a non-mathematical sense.

[3] In Chwistek's terminology as a series of expressions.

in the series and less than the largest number in the series. Chwistek sees no reason why the physicist should regard the average of these numbers as the " true " length of the table since more accurate results can be obtained by utilizing the inequality which stipulates the range of variation of the numbers obtained by measurement. For this reason he insists that the length of the table " corresponds "[1] in some way to *each* of the numbers obtained in the process of measurement and that slight differences between the results of measurement can be disregarded provided that two numbers are designated between which all the numbers obtained by measurement can be found.

Chwistek's Conception of Space and Time

Chwistek's concern with the methodological problems of physics leads him to examine the foundations of geometry. He defines geometry as an experimental science, which depends upon the measurement of segments (217), i.e. of distances. He does not, however, consider the physical procedures involved in making measurements,[2] but confines himself almost exclusively to a consideration of the logical difficulties encountered in the development of geometry as a purely mathematical science. Chwistek's considerations therefore bear upon matters which are of primary concern to the mathematician rather than the physicist, and he regards the decision between euclidean and non-euclidean geometry as not raising any problem for physics.[3] Since the constructions

[1] The meaning of the word " corresponds " in this context is not indicated.

[2] In Chwistek's elementary considerations his analysis of arithmetic assumes that the reader is familiar with the procedure of measuring segments (70). Chwistek himself has never analysed this procedure.

[3] Chwistek disagrees with Poincaré on this subject largely because the problems involved must be approached from the point of view of the physicist rather than from the point of view of the pure mathematician. Poincaré has recognized this fact but Chwistek treats the issues involved from the point of view of the mathematician. In any case the main difference between Chwistek and Poincaré with regard to the " conventional " aspects of this problem may be summarized as follows : Chwistek sees no problem at all in connection with the application of geometry in physics, because of the identity of euclidean and non-euclidean geometry when applied to limited areas ; Poincaré insists that whether one applies euclidean or non-euclidean geometry in the solution of a definite scientific problem depends upon which system of geometry is most convenient.

Chwistek opposes Poincaré's " conventional " resolution of this issue mainly because he feels that " conventionalism " leads to opportunism and irrational philosophical doctrines (234), particularly with regard to social problems.

of euclidean geometry are identical with those of non-euclidean geometry within sufficiently small areas,[1] he sees no need to decide which geometry is to be applied to existential material. In a letter dated 20th July, 1939, he says, " If we speak about a part of space there is no problem at all as to what space is to be assumed, because all spaces are approximately euclidean . . . sound reason does not countenance ideal objects, conventions, and fictions." In view of this statement it would seem that Chwistek feels either that the construction of a non-euclidean geometry is an interesting mathematical exercise with no relevance for physics, or that the application of either euclidean or non-euclidean geometry in physics yields essentially the same results. Yet he also realizes some of the difficulties encountered in applying euclidean geometry to certain portions of physics. He points out, for example, that it is difficult, if not impossible, to give an actual illustration of parallel lines. To an observer stationed at some point of a railway track, the rails seem to meet somewhere in the distance. Although it is known that they never do meet it is impossible to directly experience this fact because the observer can never simultaneously be at the point of observation and at the " point of intersection ". Thus euclidean geometry does seem to involve " fictional " objects after all (xxiv, n. 1). It is also possible to raise similar difficulties in connection with the application of non-euclidean geometry in physics,[2] since it is impossible to give an example of an experiential point or line.

Apart from his position concerning the applicability of geometry to existential material, the point which Chwistek emphasizes most strongly in his discussion of space is that

He fails to realize that all conventions are not arbitrary and that what Poincaré has in mind when he speaks of the importance of " convention " is practical convenience.

Moreover Chwistek's failure to provide a place for macroscopic objects in his theory of plural reality leads him to identify natural reality with physical reality, i.e. to treat persons and planets in exactly the same way as he treats atoms. Chwistek commits here an error analogous to that of certain nineteenth century philosophers, who, on the basis of the laws of mechanics, proceeded to argue the question of free will.

However it should be noted that while in *The Limits of Science* Chwistek opposes Poincaré's position, elsewhere he points out the importance of Poincaré's analysis. Cf. *U.B.*, p. 4.

[1] This is a *mathematical* fact.

[2] Nevertheless Chwistek recognizes that many contemporary physicists do use a geometry in which there are no parallels. He recognizes the important role of " congruence " (234–5) in geometry, although he emphasizes the mathematical definition of this concept, rather than the physical process of measuring distances.

events are specified in physics by means of a co-ordinate system. But he realizes that this specification of the spatial co-ordinates of an event must be supplemented by the specification of a temporal co-ordinate as well. There is no " real " or absolute time, but only events which are verified by reference to experience. Experience teaches us that certain events are earlier and others later, and enables us to correlate (with the help of clocks) numbers with temporal events (238). The concept of time is thus an abstraction from experience which utilizes the apparatus of mathematics. In consequence, although the physicist employs the concept of continuous time in spite of the fact that we do not experience sensibly continuous time, actually " it is impossible to take sensual continuity seriously especially because the meaning of this concept is not known " (240–1).

In his discussion of time Chwistek also points out the necessity of reasoning in conformity with a theory of types. While an individual speaking about time must speak in time, Chwistek recognizes the importance of distinguishing these two uses of the word " time ". In other words the fact that the time *in* which an individual is speaking is of a different type from the time *about* which he is speaking must be taken into consideration in all discussion.

The Philosophy of Science and the Theory of Plural Reality

Chwistek's theory of plural reality is important in connection with many problems of the philosophy of science. It is therefore worth while to give at this point several illustrations which show the relevance of this theory to certain of these problems. The physicist's concern with the problem of motion leads him to study the motion of those bodies which Chwistek calls " things ". For this reason when the physicist is studying the motion of bodies along an inclined plane, his analysis depends upon the concept of natural reality. Similarly, since the physicist interested in the motion of atomic particles has incorporated some of the results of this kind of investigation into the kinetic theory of gases, Chwistek would maintain that this theory is based upon the concept of physical reality. Even the concept of the reality of images is of importance in physics because the use of a microscope depends upon some " image " in the mind of the observer, i.e. some mental picture of that which he hopes to see with the help of the microscope (286). Chwistek also points out that the concept of the reality of sensations has been used as the basis for the criticism and re-examination of the meaning of certain fundamental concepts of physics. In this way certain " idealistic " elements found in this theory have been eliminated. He has in mind here the re-examination to which such concepts as " position " and " momentum " (256–8, 283) have recently been subjected.

The Problem of Determinism

Chwistek has been concerned with the " problem of determinism " over a period of years. *The Limits of Science* contains the fullest statement of his views on this subject up to the present time. He is interested mainly in those aspects of the problem which concern physics. Accordingly, he recognizes the futility of attempting to apply his results to the " philosophical " problem of free will. Nevertheless his discussion is marred by such digressions as his attempt to introduce Fermat's last theorem as evidence for the deterministic point of view (254).

Chwistek begins his discussion of the problem of determinism by a criticism of the " classical " views on the problem. He points out that those " classical " physicists, who advanced a deterministic conception of the world based their views on the conception of an " ideal " reality. They therefore distinguished the " real " length of a segment from its actually measured lengths. In opposition to these scientists Chwistek maintains that the notion of successive approximations to this " real length " is unnecessary, since the length of a segment can be stipulated as a number to be found between two fixed limits. Chwistek also criticizes the identification of the concept of *determinism* with the concept of *predictability*. He maintains that in a given system, even if all the elements necessary for the prediction of an event were known, it would be impossible to predict all possible results which can be obtained by the application of the rules of this system. Nevertheless Chwistek holds that this system may be determined (260) and that it may even be possible to predict the particular event in question. He offers his system of semantics as a model of a determined system, in which, nevertheless, it is not possible to predict all the results which can be obtained by the application of its rules. Thus on Chwistek's view determinism must be distinguished from predictability. For on the one hand an event may be both predictable and determined ; on the other hand it may merely be determined. It should be noted here that Chwistek employs the concept " determinism " as a primitive or undefined idea. He does not even define it implicitly with the help of a set of axioms.

Chwistek raises similar objections in connection with the views of the " classical " indeterminists. He accuses them of maintaining that it is impossible to obtain increased precision by successive approximations to the " real " length of a segment. Consequently it is not surprising that he rejects this " idealistic " conception.

Nevertheless Chwistek also rejects the view that *contemporary* physics is based upon indeterministic concepts. He explains the position of contemporary physicists on the problem of determinism as a reaction to the two " classical " (" idealistic ") views on this problem. He maintains, for example, that the criticisms of " classical " determinism do not establish the fact that a determinism without " idealistic " suppositions would in any way be objectionable (256–7). Consequently he sees no reason why physics cannot be based upon deterministic concepts. He maintains that such concepts as " position ", " length ",[1] etc., can be determined *on the basis of experience* within " sufficiently narrow limits " (258). Moreover entire *classes* of numbers, not particular numbers, satisfy the inequations which define these limits. In spite of the fact that Chwistek does not regard predictability as the defining characteristic of determinism, he maintains that only events which lie within these limits can be predicted. He asserts that if some day it should become possible to go beyond these limits the concepts " position ", " length ", etc., would acquire a new meaning, i.e. they would become new concepts for which different limits had been specified. This point is explained by means of the theory of plural reality. It becomes possible to go beyond the limits originally defined on the basis of experience, only if the process of formalizing reality [2] is taken a step further, i.e. if a new formalization of reality is set up. New concepts are thus defined with the help of this new formalization.

[1] It should be noted that these concepts have an experimental but not an absolute meaning.

[2] It should be recalled here that a " mathematical concept of reality " is involved here (xlix).

The Problem of Induction

Chwistek concludes his discussion of the methodological problems of science with a brief consideration of induction. With the help of several illustrations he makes a number of general observations concerning the nature of inductive reasoning, and enumerates some of the difficulties encountered in formulating an adequate theory of induction. Examples of unwarranted extrapolations make evident the fact that a factor relevant in one context where induction is applied, may be irrelevant in another. Furthermore no general rule can be given concerning the number of instances necessary for a valid inference to be drawn from them. In some cases inductive reasoning cannot be performed ; in others a general conclusion can be inferred from a single event. Chwistek also points out that the validity of inductive reasoning depends upon the meaning of the concepts involved, and upon knowledge of the facts and of changes in the relation between them. Moreover the elements of guessing and emotion cannot be eliminated from inductive reasoning.

Chwistek, unlike most contemporary thinkers, is not interested in justifying the use of the inductive method. He is not, however, content merely to make such general comments as those enumerated above. Accordingly, he turns to the question whether or not it is possible to construct a general law or pattern for all inductive reasoning. The discovery of the answer to this question he regards as the real " problem of induction ". He goes on to reduce this problem to the question whether or not reality can be completely formalized, in as much as his original problem and his new formulation of it both obviously involve the abstraction of certain elements from experience. Since, however, reality can never be completely formalized (261, 269), he concludes that it is impossible to give a general pattern which governs inductive reasoning. He also uses the increase in factual knowledge, which results from the extension of the scope of our experience by means of improved apparatus (266), as additional evidence in support of his position. It is thus clear why inductive reasoning can never be formulated in a pattern.

Chwistek maintains that the principle of *complete* induction, introduced as a rule of procedure in semantics, cannot be regarded as a rule of procedure in the natural sciences, since it is impossible to determine with any accuracy the " transition from any one case to the following ". Since the application of complete induction is a characteristic feature of some of the natural sciences Chwistek proposes to justify the use of this principle by means of probability considerations (267); for he sees no problem in applying the calculus of probability to existential material (255).[1]

Chwistek's views concerning the methodological problems of science were developed in consequence of his aversion to the presence of " metaphysical " elements in science. He was therefore led to formulate some very unusual views on the philosophy of science, some of which require further development. His conclusions concerning the " problem of induction " are almost entirely negative. Unfortunately he does not consider any of the specific issues under contemporary discussion concerning the validity of inductive reasoning. It would seem, however, that the very fact that no general pattern can be given for this type of reasoning should make these issues even more acute for Chwistek than for other logicians, since he has attempted to apply a single method to all portions of philosophy and science. It should be noted finally that Chwistek has made no attempt to give an exhaustive treatment of the methodological problems either of science in general or of physics in particular.

[1] The consideration of these claims must be omitted, in part because of the technicalities involved and in part because of certain unclarities which result from the fact that this portion of the text was written in English. It has proved impossible to clear up these obscurities at the present time.

This page intentionally left blank

CHAPTER I

INTRODUCTION

1. We are living in a period of unparalleled growth of anti-rationalism. Exact thinking based upon the principle of consistency is the sacrificial goat to which all the disasters of our times have been imputed. The World War and all the orgies of domineering capitalism have been attributed to rationalism developed in accordance with the requirements of exact thinking. Exact thinking is blamed for drying up the sources of the sacred enthusiasm and for causing the emotional exhaustion of our epoch. Exact thinking, it is alleged, has become the source of the excessive growth of materialistic culture, as well as the shrinkage and sterilization of spiritual life. The demand for a new logic, for new laws of thought more suitable to the needs of spiritual life, has become the hobby-horse for a whole galaxy of obscure and false doctrines, from the revived dialectic of Hegel to pragmatism, universalism, and the phenomenology of Husserl.

These doctrines have arisen in many cases owing to wide-spread ignorance, while at other times they have been dictated by completely dishonest tendencies. Their source is the tragic disintegration of science over a period of years and the despair born out of a perception of the weakness of scientific procedure.

The history of the spiritual culture of mankind may be reduced to the struggle between faith in the creative power of exact thinking on the one hand, and doubt and powerless self-humiliation in the face of the irresponsible aberrations of fanatics who never attempt to solve any concrete problem and relinquish the pleasure of overcoming real difficulties on the other hand. This struggle has been carried on for centuries with varying fortune. But at present we have entered into a period of incredible abasement of science, a period of the noisy superiority of groups of puffed-up eulogists of irrational nonsense, who are leading mankind toward open crime and violence—as a rule unknowingly but often quite consciously.

2. Reflecting on this sad state of affairs, Professor Władysław Natanson writes :

" Ought we not to regard it as evil for comprehensive science to give instruments of incalculable power to nations who have not grown up to them morally? We have conquered the forces of nature but we have not conquered ourselves. As a consequence myopic egoism arises, and as usual disasters ensue; we are retarded, we are turned back. In silence with apparent equanimity science betrays its high mission. Science has much to say to the nations. When will it say it? When will it find inspiration and power enough to warn, to restrain, to convince?" [1]

I think that one cannot leave these disturbing questions without attempting to answer them. One must at any cost ferret out the source of the evil, reveal it in all its nakedness, and completely root it out.

It will be seen that the matter is much clearer than would appear on the surface for basically nothing but fear and general inertia prevent the solution of these problems.

3. Despite all efforts, inherited prejudices concerning the metaphysical foundations of science have not as yet been overcome.

The critical attitude, with which laymen credit great scholars is not sufficiently far-reaching. When Bruno Winawer, the author of many comedies, derided the philosophers, he contrasted them with the representatives of the exact sciences and called the latter creators of new forms of life on earth.[2] He did not, however, observe that these same scholars humble themselves before philosophers and desire at any cost to set themselves up as specialists who discover the bases of philosophy.

Winawer could maintain this view so long as he did not read the popular lectures of Schrödinger, the creator of the wave theory.[3] On reading them, he was startled by the mass of irresponsible phrases and crude analogies contained therein, which compared contemporary physics with the so-called new reality in art, and the electrons with separate human individuals. He became even more disturbed when I assured him that this is a common fact and similar cases can be cited by the dozen.

4. It is a fact that naturalists of the extremely critical type restrict themselves too often to detailed investigations in their own field and disregard the endeavour for a rationalistic view of the world. Naturalists have a peculiar foible: they

[1] Władysław Natanson: *Porządek natury* (*The Order of Nature*), Kraków, 1928, p. 159.

[2] Cf. his articles published in *Wiadomości Literackie* (*Literary News*).

[3] Erwin Schrödinger: *Science and the Human Temperament*, translated by James Murphy and W. H. Johnston, New York, 1935.

indulge in metaphysical prejudices and seek popularity in the name of doctrines which go far beyond the bounds of sound reason and exact thought. Unfortunately they have great influence.

Things have come to such a pass, that to talk to-day about the distinction between the representatives of pure science and the metaphysician is indeed difficult ; for in the writings of famous mathematicians, physicists, astronomers, and biologists, abject surrender to the authority of deplorable and fruitless metaphysical endeavours is found. I shall give the following examples :

The famous German mathematician, Hermann Weyl, prefaced his book entitled *Raum, Zeit, Materie* (a work in which he endeavoured to include Einstein's theory in his system) with a philosophical introduction typical of a professional metaphysician of inferior quality. In his opinion it is a sad necessity that philosophy oscillates from system to system—a sad state of affairs which " we cannot dispense with unless we are to convert knowledge into a meaningless chaos ".[1] In other words, bearing in mind the tragic maxim, " All beginnings are obscure," [1] we are forced to build on uncertain foundations.

In the entire conception of the foundations of science offered by Weyl, one finds no trace of that modesty and unpretentiousness in the presentation of a theory which is worthy of a representative of the exact sciences. There is no recognition of that fundamental principle, that the point of departure in constructing a world view should not be a confused metaphysics, but simple and clear truths based upon experience and exact reasoning. Weyl entirely neglects the fact that physical theories are pure abstractions, which one cannot even regard as images of reality and that their rule reduces to this, that they make possible the systematic classification of phenomena as well as investigations directed toward the discovery of unknown phenomena. He ignores the fact that if philosophy is to be taken seriously it must restrict itself to a critical analysis of the relation of scientific theories to experience and cannot be the basis of these theories. He does not limit his ill-timed ambitions and seek the foundations of science ; he prefers to immerse science in a chaos of paradoxes rather than to give up beautifully sounding, showy phrases.

[1] Hermann Weyl : *Space-Time-Matter*, translated by Henry L. Brose, New York, 1922, p. 10.

Metaphysical chaos marks the ideas of even those representatives of theoretical physics who consciously construct their theories from fictional elements having nothing in common with reality. They all seem to long more or less consciously for reality and they substitute their fictions in its place. Despite the explicit stipulations formulated in their introductions, they speak definitively concerning the indeterminism of the microphysical world as if this were some reality underlying the laws of physics. Thereby they operate very arbitrarily with the concept of meaning. Appealing to the fact that the smaller their error in measuring the position of the electron, the larger their error in measuring its velocity, and conversely, they affirm that under these conditions the concepts of the position and velocity of the electron has no clearly determined meaning. Often, however, they forget to add that from this point of view, the concept of the electron itself and in general the concept of the microphysical world has no determinate meaning. If then the laws of motion of individual electrons are not discussed, that is only because it is not desired to question seriously the electron fiction.

Similar misunderstandings are evident in the case of the astronomers. When they write about the expanding universe and simultaneously maintain the finitude of the universe, they approach these matters as they would the inflation of a rubber balloon. Eddington, the famous astronomer, accepts the presence within this theory of features so paradoxical that if he himself did not believe in the theory, such a belief would exasperate him.[1] There is involved in Eddington's view a very primitive realism, in which it is hard to detect a trace of those recognized restrictions upon which contemporary science is based. There is, of course, no real basis for dispute as to the legitimacy of auxiliary constructions ; a genuine basis for disagreement is derived from the dubious pretension to knowledge of the essence of the universe, conceived in the image and likeness of the objects surrounding us.

Furthermore, disregarding the fact that their knowledge of the so-called universe is very fragmentary, astronomers would like to decide the question whether there is life beyond our globe and to give it a negative answer. In short, they would like to return to medieval geocentricism.

Reflection on these matters makes it difficult not to ask

[1] Sir Arthur Eddington : *Discussions sur l'évolution de l'univers*, Trad. et avant-propos par Paul Conderc, Paris, 1933, p. 31.

on what grounds philosophy could be disregarded and science considered the source of that revivifying word about which Professor Natanson writes.[1]

As to the biologists it must be acknowledged that they are weighed down by an oppressive mass of anthropomorphisms and irresponsible anthropological discourses which supply abundant food for philosophical dilettantism.

Driesch, the well-known biologist, is of the opinion that biology cannot do without the concept of entelechy whose internal correlate is the soul. This supposition leads him to accept some basic dynamic intellectual element which can be found both in the ultimate individual elements of a manifold as well as in the manifold itself.[2] Needless to say, here is involved an irrationalism which denies that which to-day still happens to be called scientific thought.

At the present time behaviourists carry to absurdity that important truth, affirmed by Mach,[3] that the psychic states of another individual cannot be the object of direct knowledge. The behaviourists proceed as if there were no difference between the phenomena of life and those of inorganic nature. They do not take account of the fact that so-called sympathetic attitudes or their lack are but reactions to phenomena around us, and are worthy of as much consideration as is given to sight or blindness. The fact that they take no note of these reactions leads to misunderstandings, which do not increase the authority of exact science.

Let us now contrast the metaphysician influenced by Husserl, Max Scheler, with the Dutch anatomist Louis Bolk. According to Bolk, man is a degenerate monkey incapable of normal development.[4] Man is therefore said to constitute an evident negation of life and an obstacle to nature. Evidently entirely in his element, Max Scheler takes issue with this theory. He invokes the spiritual life, culture, and art. He did not observe that Bolk's specialized knowledge makes him a dangerous opponent. Metaphorically speaking, on the level of such generalities, the contents of one empty bottle can be transferred into another *ad infinitum*.

[1] Cf. 1. 2. (In cross-references to the text the number before the decimal point indicates the chapter, the number after the decimal point the article.)

[2] Cf. Hans Driesch : "Das Organische im Lichte der Philosophie," *Atti del V Congresso Internazionale di Filosofia*, Napoli, 1924, pp. 615–625.

[3] Cf. Ernst Mach : *The Analysis of Sensations*, translated by C. M. Williams, Chicago, 1914, pp. 33–4, 54.

[4] Cf. Max Scheler : *Philosophische Weltanschauung*, Bonn, 1929, pp. 35–6, 142.

5. Finally I wish to discuss the ill-considered reflections of Sigmund Freud, the eminent professor of neurology, on the theme " love thy neighbour ". While I am not an adversary of Freud and freely acknowledge him to be one of the greatest contemporary philosophers, his polemic against the commandment: " Thou shalt love thy neighbour as thyself," [1] is based upon primitive arguments which reflect a narrow, ultra-bourgeois view of the world. This is a classic example of the harmful consequences of lack of logical training. Freud accepts the method of sound reason. Unfortunately, however, it is accompanied by all the prejudices of the bourgeoisie and he neglects the fact that the limits of sound reason are much narrower than it would appear.

The principle, that it is more desirable for an individual to have than to give, is not as evident as it seems. It may be that extreme altruism, which depends upon making sacrifices for others, is a disguised form of egoism. However, this egoism differs so fundamentally from trivial egoism that to reduce both types to the same instinct must be regarded as an obvious excursion into metaphysics, worthy of Hegel or Bergson.

When this type of discourse is compared with a work of any philosopher in the tradition of positivism,[2] it must be admitted that it is the professional philosopher who manifests clear thought and a critical attitude. It must be admitted that in our day, the great tradition of the exact sciences has ceased to be dominant and that it has become difficult to establish the boundaries between these sciences and irrational error.

Undoubtedly terms differ in meaning in different contexts. It is also difficult to extract from the chaos surrounding us that which really deserves to be called pure science. But it does not follow that it is necessary to succumb to the lure of verbal phrases because they guarantee an apparently unified view of the world. It must be explicitly noted that metaphysics is not and cannot be a view of reality because it involves a fundamental error at its very root, namely the assumption that there exists knowledge other than that which is based upon experience and exact reasoning.

Historically the desire for such knowledge has always appeared when the great aims of rationalistic science have been

[1] Cf. Sigmund Freud : *Civilization and its Discontents*, authorized translation by Joan Riviere, New York, 1930, pp. 81–7.

[2] Cf. Eugène Dupréel : *Traité de Morale*, Travaux de la Faculté de philosophie et lettres de l'Université de Bruxelles, t. iv, Bruxelles, 1932.

wrecked on a reef of paradoxes. Actually these paradoxes indicate paths toward new discoveries and toward a new and more profound formulation of scientific questions. But the dullness and inertia which are characteristic of the human mind and which lead toward the easy path of anti-rational subterfuge have permitted neither enthusiasm nor creative effort.

Those representatives of science who have not lost faith in it have concerned themselves too little with the disturbing phenomena of life about them. Alarmed and uneasy they have confined themselves to a limited sphere of detailed investigations. As often as the scientists have prepared to dictate the laws of science to mankind, they have encountered internal contradiction. Confused and broken they have been forced to withdraw from the field of strife.

There have been many such gaps in the development of scientific thought. I mention only the discovery of the incommensurability of the side of a square with its diagonal, the paradoxes of Zeno of Elea, the Copernican system, the theory of gravitation, the critique of pure reason, non-euclidean geometry, the theory of Darwin, the paradoxes of the theory of aggregates, the experiment of Michelson, and Einstein's theory which is based upon it, the discovery of radium and of the quantum properties of radiation. On each of these occasions the scientists, dismayed by their results, retreated and left the fate of human culture to the discretion of individuals, who were unable to perceive the implications of these findings and therefore were inadequately equipped to carry on the struggle. It was at just such a time that Plato rose, and on similar occasions Hegel and later Nietzsche, Bergson, and their lesser followers achieved extraordinary success. They gave to disappointed mankind a narcotic of vision and phantasy which they substituted for exact thought. Society prepared to yield to the authority of chosen intellects and to abandon its critical attitude toward prevailing relationships. I do not deny that on occasion their ideas were worthy of admiration, yet I cannot refrain from pointed criticism of their procedure.

6. Rational criticism originated in Greece, but because it had too radical a form in its early stages, it led to extreme scepticism. The Greeks were unable to surrender their over-extensive epistemological ambitions, and therefore easily succumbed to a feeling of despair during periods of failure. The paradoxes of Zeno of Elea are known to have checked the

development of Greek mathematics. Similarly the discoveries of Heraclitus and Protagoras, which are correct in principle, are known to have completely checked belief in a science based upon experience.

To-day it is difficult to evaluate the social influence of the sophists, because their activities are known only from the writings of their admitted enemies. Nevertheless it is certain that they helped unmask false methods of reasoning, current in societies supported by tradition and the authority inseparably connected with it. It is well known that in such societies, the most diverse absurdities are held to be evident merely because they have been so regarded for generations. Sound reason is identified with the cultivation of these absurdities. Any opposition is held to be a bad error.

Undoubtedly the sophists went too far in their criticisms; nevertheless it must be pointed out that the Greek scientists could not refute their doctrines because they neither possessed the principles of exact thought nor knew its limits.

Socrates opposed the sophists in the belief that there exists a sphere of thought independent of our caprice; however, its domain was the world of universal, confused, and vague concepts, i.e. precisely those which even to-day are subject to individual interpretation. Consequently this belief was inevitably doomed to defeat.

Lev Shestov emphasized that great truth which was known to several earlier authors, that the arguments employed by the platonic Socrates are both cavilling and subjective. Yet in contrast to the arguments employed by the sophists they are held to be objective and absolutely true.[1]

Plato was undoubtedly a creative individual of the highest rank. But that which has always been regarded as his chief merit, namely the fact that he avoided arbitrariness and subjectivism through the use of universal concepts, was also his greatest error and the source of long-lasting stagnation.

Centuries elapsed before man rediscovered the proper province of exact reasoning. During this period, from the famous Plotinus to Hegel and the pragmatists, the platonic dialectic was the source of the mythology imposed upon objective science.

Yet it cannot be denied that it was Plato's disciple, Aristotle, who took the first step toward constructing a system of the

[1] Lev Shestov: *Le pouvoir des clefs*, translated by B. de Schloezer, Paris, 1928, p. 101 ff.

principles of exact thought. Aristotle's system was a fragment and was not free from fundamental errors. It was but the first step along a long and wearisome path. Unfortunately men have regarded Aristotle's work as perfect and for many centuries did not attempt to go beyond it. Even to-day it is still defended and passionately adhered to, although it is well known that its study is a useless requirement. For many centuries learned theologians imposed upon mankind as valid truths strictly proved by means of Aristotle's infallible system, doctrines which have nothing in common with exact reasoning. Actually they more or less consciously made the most of the obscurities and defects of this system.

Nevertheless the postulate of consistency, formulated by Aristotle as the well-known principle of contradiction (*Principium contradictionis*) [1] was a great triumph of rational thought.

It is possible to try to avoid this principle and to extricate oneself from the maze of contradictions which result from false assumptions motivated by utilitarian considerations. However, no one has had the courage to say that better reasoning exists, which need not be governed by the principle of consistency. Furthermore, this principle permits the discovery of errors in reasoning where lawlessness reigns.

The Russian philosophers of the Bukharin school condemn Aristotle's logic because it permitted the fiction of immutable concepts and supported the existing social order and the blemish of slavery.[2] They credit Hegel with having unmasked the prejudice concerning the immutability of concepts and with having opened the way to social progress.

This entire doctrine was caused by misunderstandings.

The postulate of consistency created a basis sufficient to overthrow a social system based upon slavery. The institution of slavery involves obvious contradictions, although the attempt is made to conceal them by more or less skilful phrases. On the one hand slaves are regarded as beings inferior in principle and essentially different from free men ; but on the other hand the right to sell free men into slavery is accepted. Aemilius Paulus abandoned Epirus to his mercenaries and sold one hundred and fifty thousand free men into slavery. Among

[1] Cf. Aristotle : *Metaphysica*, translated by W. D. Ross, Oxford, 1908, bk. iv (Γ) ch. 3–6, 1005*a*–1011*b*.

[2] Cf. Nikolai Bukharin : *Historical Materialism*, authorized translation from the third Russian edition, New York, 1925, pp. 23, 170.

those enslaved were keen, sensitive men who were highly developed mentally and ethically. The moment they were enslaved they became chattels subject to the unrestricted orders of their owners, who in many cases ruthlessly and cruelly took advantage of their rights. The money obtained was divided among the soldiers. Each soldier obtained no more than eleven drachmas. Plutarch wrote :

> " Men could only shudder at the issue of a war, where the wealth of a whole nation thus divided turned to so little advantage and profit to each particular man." [1]

These words of Plutarch are significant. It is seen that where cultivation of the emotions is not on a sufficiently high plane, the principle of consistency does not serve as a check.

Considerably before Hegel, the prejudice concerning the immutability of concepts was overthrown in practice, if not in theory, by the rationalists of the eighteenth century who prepared the way for the French Revolution. Undoubtedly the French Revolution developed from a rationalistic culture which was based upon exact thought associated with the spirit of mathematics and disdainful of all thought that was not clear and precise.

This culture was a phase of the development of the dogmatic rationalism of the sixteenth and seventeenth centuries which was based upon the belief that mathematical and natural methods make possible the discovery of the essence of all things. This belief resulted in a series of contradictory philosophical systems and the need for a critical attitude.

The strength of the pre-revolutionary philosophers did not lie in positive constructions, but in criticism based upon the methods of the exact sciences and extended to limits not reached in any previous period. The materialists overthrew the myth concerning the substantiality of the soul. Hume unmasked the false ambitions of dogmatic rationalism. Voltaire ridiculed the false pathos of the Middle Ages, the cult of the devil, and the unbridled licentiousness of the feudal lords. Montesquieu overthrew the belief in the apriori character of law and morality by a simple comparison of facts drawn from different times and different societies. Jean Jacques Rousseau overthrew the prejudice concerning the intellectual superiority of the

[1] Plutarch : *The Lives of the Noble Grecians and Romans*, translated by John Dryden, revised by Arthur H. Clough, Modern Library edition, a reprint of Clough's edition published in 1864, New York, p. 339.

privileged classes. The Encyclopædists robbed science of the mysterious charm of the book of the seven seals.

It is known that Saint Simon, a typical rationalist who accepted the critical world view of the pre-revolutionary epoch, was the creator of socialism. The positivist Auguste Comte who was greatly influenced by Saint Simon was the creator of sociology. Hegel's dialectic was not necessary to see the inconsistency of a social structure which was characterized by the oppression of the poor classes by the ruling class. The Aristotelian principle of consistency and cultivation of the emotions, based upon the doctrine of Christ, was quite sufficient for this purpose. That Karl Marx, the great defender of the wronged, was a disciple of Hegel and appealed to Hegel's dialectic was the result of a fortuitous concurrence of circumstances. Hegel's dialectic was a minor influence in the works of Marx and did more harm than good because it produced the illusion that Marx was concerned with the self-contradictory idea. Actually Marx was concerned only with the distress of the working-class and with the creation of a view of the world which would remove this distress. The dialectic of Marx does not differ essentially from the constructive methods of the exact sciences. Marx did not write on the " love thy neighbour " principle only because even at that time this principle was an outworn requisite, which could not restrain bestialized business men.

The positivistic doctrines of Auguste Comte strongly influenced nineteenth century scholars and produced a type of critical investigator who, like Newton, rejected such questions as : what is this ? and why is it so and not otherwise ? —because such questions lead to fruitless investigations. This type of investigator tries rather to extend the bounds of experience as much as possible and to formulate the laws governing phenomena. This doctrine, although radically altered during the course of the years, is maintained to this day. Thanks to the works of Mach, Poincaré, Einstein, and many other investigators has been produced the contemporary naturalistic view of the world, based upon the principle of the economy of thought formulated by Mach,[1] and the relativity of the principles of the theoretical sciences. It might be thought that these doctrines would lead to the victory of the ideas of tolerance and social justice. Actually this did not

[1] Cf. Ernst Mach : *The Science of Mechanics*, translated by Thomas J. McCormack, Chicago, 1902, pp. 6, 480–490.

happen. In this view of the world there remained certain weak points, against which occurred an anti-rational reaction leading to the present sad state of affairs. The source of this reaction should now be sought and definitively eliminated.

7. Hegel was undoubtedly the creator of contemporary anti-rationalism. His doctrines resulted from misunderstandings which were caused by a superficial knowledge of Kant's philosophy and a completely erroneous conception of mathematical analysis. Kant contrasted metaphysics, whose concepts necessarily lead to paradoxes, with mathematics, the sphere in which precisely defined concepts are employed. Kant's doctrine became a permanent acquisition of mankind. However, a century of work was necessary before it was understood that the primitive concepts of mathematics are as variable as other primitives and that their force lies only in the fact that their domain can be fixed by means of a precise symbolism. Hegel made this weak point in Kant's doctrine the basis of his dialectic. Hegel seized upon the fact that the mathematical analysis of the time depended upon the very confused concept of infinitely small increments or infinitesimals. He argued that there are and there are not infinitesimals ; therefore infinitesimals are undoubtedly contradictory objects.

On the other hand it is a fact, Hegel maintains, that mathematical analysis permits the discovery of the laws governing nature, while a consistent algebra is on the whole quite unproductive. It clearly follows that mathematical analysis owes its fruitfulness to the contradictions contained within it. But if such be the case, the fact that metaphysics leads to contradiction is not a defect but the guarantee of its creative power. Let us therefore cease to fear contradiction. Rather let us be courageous enough to accept its aid. We will then discover the essence of things which Kant had regarded as unknowable, namely the creative idea which contradicts itself at every step.

This view is obviously much more presumptuous than any view hitherto developed by mankind.

Paradoxes which from the beginnings of the history of intellectual culture had been regarded as fatal, suddenly became the source of great optimism and an instrument to be employed in obtaining knowledge of absolute truth. These doctrines offered so great a temptation that it can hardly be wondered that they gained many adherents and are found to-day in many different forms. I think that the fundamental duty of the representatives of true science is to combat this doctrine.

The style in which Hegel addressed his students clearly reflected popular mysticism. It might have been thought that a practical joker had cut parts of sentences out of scientific works and put them together arbitrarily in order to stupify and frighten mankind. Because of his great knowledge of human nature and his great talent for propaganda Hegel knew how to insert whole paragraphs amidst a mass of nonsense and pretentious poppycock.

He begins with the idea that to be and not to be is the same thing, whereupon the following explanation is added:

> "If we look more closely we find that a proposition has here been asserted which, carefully considered, has a movement by which through its proper nature, it disappears. But in so doing it does what must be held to constitute its true content, it undergoes Becoming." [1]

When the reader is convinced that he can never fathom the thoughts of the master and that he can only gather crumbs which are gratuitously thrown to him, he encounters the following doctrine:

> "God is known as Spirit, who duplicates Himself for Himself, but at the same time sublimates this difference in order that in it He may be in and for Himself. It is the task of the world to reconcile itself with Spirit." [2]

By this time the reader begins to realize that the master has disclosed to him the proper goal toward which he should strive at all cost. Unfortunately he does not know how to do this. He feels that he lacks courage and has no confidence in his own powers. The master then addresses him in an entirely new language, a language which is wholly colloquial, in fact just such a language as is employed by every cook and provincial shop-keeper. He says:

> "... this task is entrusted to the German world." [2]

A new epoch in philosophy began with Hegel and the German romanticists. It was characterized by the exploitation of the vagueness of concepts for purposes of the authors, the utilization of a conception of the universe in the interests of aristocratic reaction and the conscious use of this conception for

[1] G. W. F. Hegel: *Science of Logic*, translated by W. H. Johnston and L. Struthers, New York, vol. i, 1929, pp. 102–3.

[2] G. W. F. Hegel: *Vorlesungen über die Geschichte der Philosophie*, edited by G. J. P. J. Bolland, Leiden, 1908, Einleitung, p. 85.

cheap rhetorical effects. Schopenhauer vainly tried to unmask Hegel's method. But his voice was drowned by the plaudits of over-enthusiastic admirers. A phalanx of imitators soon arose. Hegelians appeared in almost all the countries of Europe and it is well known that all Polish philosophy during the first half of the nineteenth century developed under the influence of Hegel.

The plague of anti-rationalism cast into the world by Hegel spread to fantastic bounds. At first it was stifled by the tremendous growth of the mathematical and natural sciences; later as a result of the crisis in these forms during the nineteenth and twentieth centuries, it assumed new forms.

The first of these new forms was Friedrich Nietzsche's artistic conception of reality. Soon afterwards Bergson suggested the re-examination of the paradoxes of Zeno of Elea from which resulted emphasis upon a new faculty of knowledge called intuition. Bergson was influenced by the monstrous doctrine of the pragmatists. At the same time there began to spread in Germany the "scientific" philosophy of Husserl which attempts to create a new scholasticism in the name of the belief in absolute knowledge.

After the war, Europe was swamped by vast numbers of irresponsible anti-rationalistic systems which in the main yielded nothing new but were based upon cheap phrases which have been known for a long time. These systems helped create an atmosphere of depression and fear among the representatives of the exact sciences and had a more or less conscious influence upon their method of thinking.

On reading the arguments of Bergson, it should be observed that they are based upon a number of unresolved mathematical problems. While Bergson condemns the mathematical method and seeks to replace it by a better method he characterizes the latter with the help of mathematical concepts which are awkward and colourless.

Bergson relies upon *vague* and *variable* concepts as symbols of acts of a special kind. While these acts cannot be performed, Bergson perceives their possibility intuitively. Intuition is not to be identified with that "experience which arises from the immediate contact of the mind with its object, an experience which is disarticulated and therefore most probably disfigured".[1] Intuition is true experience, an experience which

[1] Henri Bergson: *Matter and Memory*, translated by Nancy M. Paul and W. Scott Palmer, New York, 1911, p. 239.

is higher than that which is called human experience. To have this experience it is necessary to give up certain habits of thought and even of perceiving and to place oneself at the *turn* of human experience. Bergson writes:

> " . . . there still remains to be reconstituted, with the infinitely small elements . . . of the real curve, the curve itself. . . . In this sense the task of the philosopher . . . closely resembles that of the mathematician who determines a function by starting from the differential. The final effort of philosophical research is a true work of integration." [1]

Because mathematics has not satisfied any of our desires, which in fact cannot even be formulated, it should be thrust aside. Instead of mathematics a new science which will be easier and more pleasant will be created. In lieu of struggling with the difficult problems of the theory of mathematical functions more attractive operations will be performed which meet no opposition, because no basis for opposition can be found. Anyone of a different opinion may be told that he is lacking in intuition and that he must feel his way intuitively. After a certain time he will either understand or pretend that he understands. In this way will be revived the tradition of the old magi, who held society by dangling before it secret and esoteric knowledge. Intellectual slavery from which mankind cannot escape is to be organized on a large scale.

These or similar thoughts must have been present in those minds which are excessively individualistic and which, influenced by Bergson, are inclined toward opportunism. They appeared openly and shamelessly in Anglo-Saxon countries as pragmatism (William James) and humanism (F. C. S. Schiller).

Both these doctrines skilfully exploited the positivistic conception of the economy of thought (Ernst Mach) which in conformity with the views of Bergson, they grotesquely caricatured.

The principle of the economy of thought [2] cannot be precisely formulated. In practice it reduces the aim of science to the construction of a theoretical system of concepts which would enable us to know the truth, i.e. to adjust ourselves in the real world as simply as possible. The theory of Copernicus may be cited as an example. However, actually the theory of Copernicus decided nothing because it can be maintained with equal

[1] Bergson: *l.c.*, pp. 241–2. [2] Ernst Mach: *l.c.*

right that the earth revolves around the sun and that the sun revolves around the earth. This latter view follows from a consideration of the relativity of all motion. Nevertheless the theory of Copernicus permitted us to describe in simple fashion the motion of the planets and later led to the Einsteinian principle of relativity.

Neither can the latter principle be regarded as an absolute truth, although it permits the construction of a much more unified cosmological system than the Newtonian system, and although it discloses phenomena never dreamt of before. As a whole the system of Einstein is much more complicated than that of Newton, but it is much more economical in dealing with the problems which present themselves. It is therefore seen that naïve simple economy is not sought for its own sake, but as a means for the attainment of knowledge of that which is before us.

The pragmatists did not take the trouble to think these matters through. All they said was that if economy is being discussed, value and therefore utility, and not truth is concerned. There is no concept of truth without the concept of utility. That which is useful is true. Neither pure thought nor pure knowledge exists. Always and everywhere the element of belief and individual want is decisive. Concern with matters which are not connected with life is fantastic.

The reality of every day is not true reality, writes F. C. S. Schiller.[1] True reality is created in accordance with one's needs. There may be a certain hesitation in accepting this theory because the supposition of the reality of the things and persons surrounding us is regarded as useful. However, if it is understood that the belief in the ability to create a new reality is much more useful, no hesitation with regard to accepting this belief unconditionally will occur.

According to Schiller, Protagoras was the real creator of humanism. Schiller does not interpret the doctrine of Protagoras as extreme scepticism but as a new type of metaphysics.

Schiller treats the difficulties which arise in connection with the fact of the existence of mathematics. He briefly remarks that the evidence and objectivity of mathematics is an illusion which results from familiarity with the postulates of mathematics, the frequently accepted belief in their practical importance and the fact that mathematics does not consist

[1] Cf. F. C. S. Schiller: *Studies in Humanism*, London, 1907, pp. 220–1.

of a series of isolated truths but its truths form a unified and coherent system.

Schiller does not mention the fact that such truths, as twice two is four, cannot be denied without considering the system of which they are a part, although in many cases it might be to our advantage to do so. Certainly everyone would deny that there is an advantage in accepting a system of arithmetic where $2 + 2 = 5$ if he has to pay someone else, but where $2 + 2 = 3$ if he is to be paid.

Contradiction would indeed follow from this, but in many cases contradiction has proved to be very useful and often cannot be distinguished from reasoning which is based on feeling and emotion. Consequently if on the grounds of utility, mathematics is generally regarded as an objective science, perhaps by the former term is meant a utility which has nothing in common with individual criteria, a metaphysical utility which is inaccessible to ordinary intuition. But utility in this sense, in so far as it is not simply a synonym of what Schiller calls truth, is a mere phrase.

Schiller does not refrain from employing underhand demagogic tricks. He fails to consider arguments in which rational criticism is employed and deals only with the naïve idealism of certain English metaphysicians. Consequently his arguments seem ultra-intelligent and effective. It is not strange then that the superficial reader, especially one who is seeking a quick and easy solution of the problem of knowledge does not see this.

The influence of the doctrines of Schiller and James has been much greater than is apparent. I quote from them but rarely, although their ideas live on in the works of many post-War German metaphysicians and adversely influence the minds of exact investigators the world over.

It seems to me that it is a waste of time to argue with them. I think that it is sufficient to emphasize the nonsense involved in their doctrines. It is much more important to confront them with a consistent world view which has been developed by the use of a critical and rationalistic method and which involves no metaphysical suppositions. The first step toward the attainment of this goal must be a consideration of the foundations of logic and of the question whether it is really possible to construct a system of logic which involves no metaphysical suppositions.

8. The doctrines of the phenomenological school, which was founded by the late Edmund Husserl, are representative of a certain type of anti-rationalism.

Husserl did not oppose science but desired to supplement it by a scientific philosophy based upon the conviction that absolute knowledge is possible.

I quote here the criticism of the doctrines of this school which I made in the introduction to a previous essay.[1]

> " The fact that the phenomenologists themselves repeatedly show the absurdity of their belief in the possibility of absolute knowledge spurs them on and forces them to unparalleled efforts, which prevent the development of their most characteristic views and prevents their progress beyond the sphere of pleasing conventionalism. Nevertheless the sin of verbalism, sanctioned by the deplorable ' pure grammar ' of Husserl shakes this school to its foundations. Not only do the phenomenologists fail to attain the heights of which they have dreamt but they are brought back to the muddy dells of reality. . . .
>
> " Pure grammar is the means used by the naturalists and the naturalistically orientated epistemologists to prevent a critique of the bounds of everyday language. These investigators gave careful consideration to the meaning of philosophical questions which are apparently innocent and natural. Thus they initiated the exact investigations later conducted by the logicians, and in particular by the famous Bertrand Russell. Among other things they were concerned with such eternal questions as : What is truth ? What is matter ? What is man ? What is the good ? What is a work of art ? and so forth.
>
> " Those scientists who had failed to consider these questions, lost contact with reality and entered the sphere of fiction. The chief problem of the phenomenologists was to be the rebuilding of this contact."

It was to be re-established by an investigation of the real content of concepts, which does not differ essentially from that of which Plato dreamt. Different methods were applied but they were just as arbitrary and confused. Husserl began with a cavilling criticism of the nominalism of Hume[2] and pointed out that Hume did not show how it happens that certain ideas are produced by certain words. This objection is obviously childish because much more complicated phenomena are involved here than the phenomenon of gravitation, for example, about whose essence we neither know, nor hope to know, anything. Accepting independent concepts to establish meanings is very much like

[1] *T. V. M.*, pp. 48–9.

[2] Cf. Edmund Husserl : *Logisch Untersuchungen*, Bd. ii, Teil 1, Halle, 1913, pp. 184 ff.

positing that the weight of matter is located at the centre of the world in order to establish the law of gravitation.

The positive work of Husserl explained nothing because of necessity it was based upon arbitrary and confused assumptions. His works are filled with such assumptions.

It is clear, for example, that for Husserl the word *something* has a simple meaning.

> " The experience of an idea which is consummated in understanding the word is undoubtedly a construction, but its meaning is without a trace of being compounded." [1]

This distinction between the meaning of the word and understanding it is the result of verbalism and arbitrariness, since it is impossible to discover anything other than the ideas which present themselves when the use of the word *something* is being considered. These ideas might be called the meaning of this word. The phenomenologists maintain that only by the use of their hypotheses can the relativism which makes science impossible be avoided. Actually it is the phenomenological method which makes science impossible because it makes science depend upon some special faculty which has nothing in common with either reasoning or experience and which can not be controlled. If the method advocated by the phenomenologists were employed, sooner or later esoteric knowledge of the type found in the Ancient East and the intellectual and material slavery associated with it would recur. The pre-War essay of Reinach,[2] a disciple of Husserl, plainly manifested this tendency.

The Bolshevik revolution and its unexpected success destroyed the social illusions of the phenomenologists and transformed them into the obvious anti-rationalism of Hitlerian insanity.

9. The negative aspect of both positivism and materialism is that on the basis of these doctrines it is impossible to fix even approximately the boundaries of the exact sciences. In particular it is difficult to define the special status of mathematics. The old Kantian argument which depended upon the thesis that mathematical truths are certain while those of nature are approximate was until recently universally accepted. Even Poincaré took it seriously. The voices of those naturalists who

[1] Husserl: *l.c.*, p. 296.

[2] Cf. Adolf Reinach: " Die apriorischen Grundlagen des bürgerlichen Rechtes," *Jahrbuch für Philosophie und phänomenolo ische Forschung*, Bd. i, teil ii. 1913.

observed that arithmetic is not to consider the changing world (Le Dantec), had no great influence. For the most part attention was focussed on the fact that the world of geometry is an ideal world and differs fundamentally from the sensual world. The attempts of John Stuart Mill to regard points, lines, and planes as hypothetical objects proved to be unfortunate. This was also true with regard to Mach's attempt to discover correspondents of these constructions in the sensual world. It is clear that geometry does not depend upon inquiries of this kind and those who have advocated apriorism based their views upon this fact.

The reduction of arithmetic and geometry to the principles of formal logic, which was attained by Whitehead and Russell at the beginning of the century, was the crucial moment in the attempt to fix the boundaries of the exact sciences.

If the attempt to construct a great system of logic from which all the apriori sciences could be derived were successful, completely new perspectives would be opened up to science and an adequate basis for a critical and rationalistic method would be attained. A system of logic which permits mathematical theorems to be proved without the aid of the intuition of the creative individual by mechanical operations, which can be performed by any one who can understand ordinary arithmetic, was sought. The attainment of this ideal would have been so great a triumph for science that in comparison with it the attempts of the irrationalists would seem like child's play. It was to be expected that the representatives of radical criticism would have accepted the work of Whitehead and Russell with enthusiasm. However, the exact opposite actually occurred.

Peano's earlier attempt to formulate the apparatus of concepts and axioms of mathematics had already evoked a violent reaction on the part of Poincare. Actually he both feared to break with the positivistic tradition and mistrusted the reaction of an extremely critical mind toward a work which had many weak points. Peano's apparatus of concepts was still far from perfect and it was possible to ask whether it would not lose its force the moment it was desired to mechanize it completely.

The second crucial moment in the development of this line of thought was the discovery of the paradoxes which follow from Cantor's theory of aggregates. These paradoxes were involved in the foundations of the new logic. Russell succeeded in removing these paradoxes by means of his famous theory of logical types but he was able to do so only by introducing certain metaphysical suppositions which a critical mind could not accept.

In the first place it was necessary to presuppose the existence of individuals which could not be further characterized. The existence of these individuals was an integral part of the system but no example of them could be given. In other words the domain of logic became an abstract world similar to the platonic world. The primitive concepts of logic became platonic ideas because they had to be explicitly distinguished from the signs by which they were introduced. Finally it was necessary to accept an additional hypothesis which assured the existence of infinitely many individuals. Otherwise finitism could not be avoided. On the other hand, if this hypothesis were accepted the existence of objects not definable in terms of the concepts of the system would have to be accepted.

In short it must be admitted that the system of Whitehead and Russell is such that either it does not contain the class of natural numbers or it contains a class of real numbers which contains as a sub-class numbers not definable in terms of the concepts of the system. The latter consequence which at the same time leads to the affirmation of the existence of the actual infinite evoked a particularly vehement reaction on the part of Poincaré. Poincaré was a decided nominalist and could not become reconciled to the existence of indefinable objects, much less to the existence of infinite classes of such objects. Poincaré regarded his belief as the fundamental postulate of a nominalistic logic. He formulated this postulate as follows : " Consider only objects which can be defined in a finite number of words."[1]

Poincaré thought that by proposing this postulate he invalidated the entire construction of Whitehead and Russell. Most of the adherents of the new logic were of the same opinion. This fact clearly shows the extent to which science depends upon philosophic views.

Mathematicians were divided into two groups. The members of the one group called themselves empiricists and the influence of Poincare upon them is clearly observable. I think that they should be called nominalists. The nominalists rejected systematic logic, were satisfied with mathematical, intuition, and confined themselves to a verbal characterization of the intuitive method (Brouwer). This method differs from that employed in constructing a precisely defined system. The idealists, who from the point of view of the medieval tradition should perhaps be

[1] Henri Poincaré : *Dernières Pensées*, Paris, 1913, p. 138.

called *realists*, constituted the second group. The members of this group, relying upon Cantor's theory, failed to mention the system of Whitehead and Russell and restricted themselves to intuitive attempts to demonstrate the consistency of the axioms of mathematics (Hilbert).

With the mathematicians so divided in their opinion the expected rebirth of the exact sciences on the basis of a great system of logic which fixes their boundaries failed for the moment.

But the game was not finished.

Further investigations showed that the metaphysical suppositions of the system of Whitehead and Russell can be eliminated by basing the construction of a consistent system of logic upon a pure theory of types and upon the science of expressions, formulated symbolically, which I have called semantics. In other words the additional suppositions made by Whitehead and Russell are unnecessary.

Thus a new system of logic which satisfies the nominalistic postulate of Poincaré and which is compatible with the spirit of critical rationalism was developed. In spite of the extensive restrictions of this system it is no poorer than the system of mathematics which is based upon the axioms of Zermelo. Consequently it is adequate to develop all the material which is desired by most mathematicians.

When this new system is completely worked out, we will be able to say, that we have at our disposal an infallible apparatus which sets off exact thought from other forms of thought.

The old dream of the logicians concerning a consistent logical apparatus will no longer be a mirage. Just as now we have calculating machines, in time we will have the apparatus which is necessary to derive the general theorems of semantics.

However, I think that there is no reason to wait until this ideal has been achieved.

The very confirmation of such a possibility offers weapons which are adequate to combat the attacks of the anti-rationalists and to free us from any possibility of attack by them.

A science which is based upon an infallible system of logic and which involves no irrational assumptions will be able to fulfil the mission toward society which Professor Natanson requires of it.[1]

Such a science will not fall into error and will not be brought to a standstill as a result of its own illusions.

[1] Cf. 1. 2.

Such a science will be able to say to the nations :

Construct new concepts if nothing else, but guard against operating arbitrarily with them. Remember that otherwise chaos and error will result, and that it is possible to avoid them only with the help of a complete system based upon the principle of consistency.

Have the courage to search the obscure hidden corners of your system and do not be ashamed to admit that you were following the wrong path if from your assumptions you derive conclusions which contradict these assumptions. Do not believe that exact analysis necessarily leads to inertia and the depreciation of the imagination and emotional life.

The fact that recently a nation with a great cultural tradition has been mastered by brutal, ignorant individuals shows only that this nation was permeated by an irrational metaphysics.

History teaches that ultimately victory has always been the destiny of societies who employ the principles of exact reasoning Exact analysis depraves only weak and inept individuals who find it too difficult for them. It should not be feared by young and healthy societies. They will always find sufficient strength to act upon thoughts which were obtained over a period of years by means of exact analysis and to work out a well rounded fruitful life on the basis of these thoughts.